History of Mathematics and the Forgotten Century

Glen Van Brummelen

The history of mathematics is being reinvented. Over the past few decades, we have started to realize how delicate a matter it is to portray historical mathematics without distorting it with our modern viewpoints, especially if the subject is centuries old. For instance, we are now careful to avoid expressing, say,

> Elements 11.4: "If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments."

as its algebraic equivalent, $(a + b)^2 = a^2 + b^2 + 2ab$. This representation changes the impact the theorem would have had on Euclid's audience, in this case obscuring its applications to irrational magnitudes and conic sections. This new sensitivity is a good thing. History isn't about translating ancient accomplishments into modern equivalents; it's about understanding how other communities and cultures thought differently from ours.

But we still have a long way to go. There is more to representing history than our choice of language. For instance, we still often select what mathematical topics in history to study by their interest and accessibility to us. As a result, the broader mathematical community considers the story of European mathematics to be not far off from the story of the development of today's university mathematics curriculum. We talk a great deal about the origins of calculus (not a bad thing, on its own!) and the associated pivotal transition of mathematics from its Euclidean life in the Platonic realm to interactions with the physical world. We also hear of the beginnings of analytic geometry, and more recent topics such as 19th-century analysis, the rise of abstractions resulting in abstract algebra and so forth. To some extent, by what we choose to discuss, we are still talking about us, not them.

Putting this to the test: quickly, name a 16th-century European mathematician. Names like Newton, Fermat, Descartes and Leibniz roll off our tongues easily . . . but they were all active in the 17th century. If pressed, some of us might come up with Girolamo Cardano, who solved the cubic equation, or François Viète, who is associated with the establishment of symbolic algebra. But that's about it. Was the 16th century really so sparse?

In fact, there is a flourishing community of scholars who focus on this period, but their efforts have not entered easily into the popular mathematical imagination. What has filtered through, in addition to Cardano and Viète, are several contributions to the emergence of algebra and the beginning of various notations, Rafael Bombelli's early discovery of complex numbers and the invention of decimal fractions. Again, these are topics related to the modern mathematics curriculum. Behind this, the literature contains crucial developments, many yet to be discovered. Some of the most important of these should lead us to reconsider whether or not it was calculus that brought European mathematics in contact with the physical world.

My own recent readings in my research area, the history of trigonometry, have brought these points home. Of course, I may well fall victim to my own critique that we tend to focus on historical topics selected by our own interests! However, most of this story is not well known. Spherical trigonometry and early approaches to mathematical astronomy, alas, are not on everyone's lips these days. This is a clear instance where shifting tides in today's school mathematics have obscured for us significant historical events in mathematics.

The fundamental work in trigonometry of the 16th century was Regiomontanus's *De triangulis omnimodis,* written in 1464 but not published until 1533. As the title indicates, it provided solutions to all types of triangles, both plane and spherical. His purpose, as with all such writings at the time, was to provide effective tools for mathematical astronomy. In fact, he referred to his book as "the foot of the ladder to the stars." Unlike surveying, the sciences, or other applications, astronomy was considered to be a fit subject for higher mathematics. When mathematics was needed for earthly matters, more elementary tools from "practical geometry" were used.

The middle of the century saw the appearance of the six nowstandard trigonometric functions in Georg Rheticus's 1551 *Canon doctrinae triangulorum*, and in this same work, a hint of the discovery of the 10 standard identities for right-angled spherical triangles. These results appeared explicitly in Viète's first mathematical work, *Canon mathematicus seu ad triangular* (1579), where, incidentally, we first see his propensity toward symbolic representations. But still, authors' eyes were fixed firmly on the goal of astronomy.

This began to change just a couple of years later. In 1581 Maurice Bressieu hesitantly included an appendix to his *Metrices astrvnomicae* that showed how to use trigonometry to find the altitude of a castle. Just two years later, Thomas Fincke's influential *Geometria rotundi* included an entire chapter devoted to the use of trigonometry in surveying. Mathematicians' enthusiasm for these new benefits of their work continued to accelerate; Bartholomew Pitiscus's 1600 *Trigonometriae* (the first appearance of the word) lists prominently on its title page geodesy, altimetry and geography, along with the more conventional astronomy and sundials.

Around the same time, Edmund Gunter and others were building instruments, such as his quadrant and his scale, to solve problems that could be used in navigation and other practical arts. These tools became popular, but they were not immediately accepted by the mathematical establishment. Interviewed by Henry Savile for the first post of Savilian chair of geometry at Oxford, Gunter demonstrated the amazing powers of his instruments. It is reported that Savile responded, "Do you call this reading of geometry? This is showing of tricks, man!"

Broader acceptance of mathematical methods received a major boost with John Napier's introduction of logarithms in his 1614 *Miriffci logarithmorum canonis descriptio*. Napier's purpose in this work was to streamline calculations especially in spherical trigonometry, which frequently requires the multiplication of irrational trigonometric quantities. Laplace later said that Napier, "by shortening the labours, doubled the life of the astronomer." But the biggest impact of logarithms was not heavenly, but earthly. It accelerated the acceptance of mathematics by practitioners. Authors like John Norwood

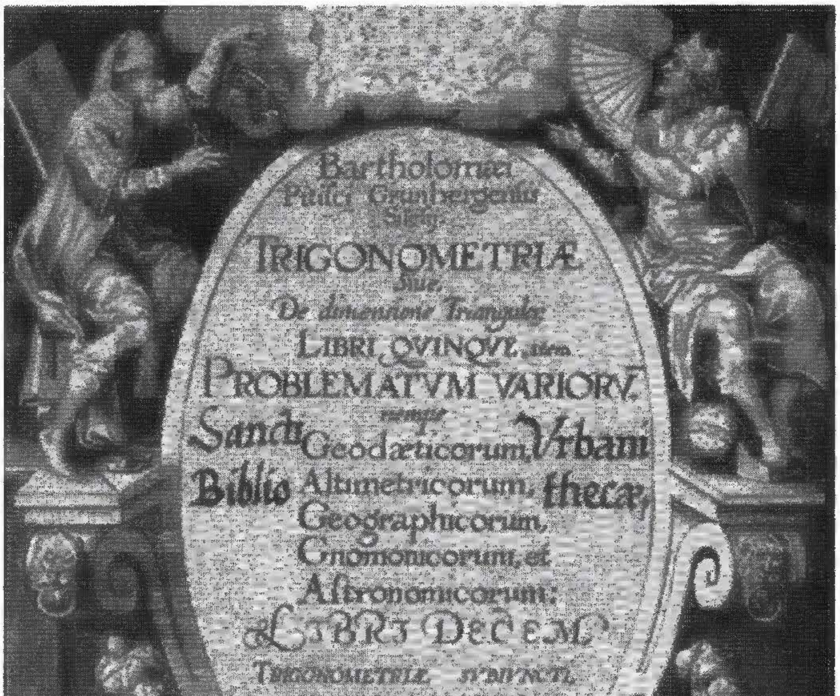

Figure 1. *Title page, Bartholomew Pitiscus,* Trigonometriae, *rev ed (1600), Rare Book and Manuscript Library, Columbia University in the City of New York. See www.maa.org/press/periodicals/convergence/mathematical-treasures-bartholomew-pitiscuss-trigonometry.*

started writing manuals demonstrating the use of the combined trigonometry and logarithms to facilitate calculations for topics such as military architecture. The world, truly, was becoming mathematized.

It was in this context that Galileo's famous quotation from *The Assayer* (1623), that the universe is written in the language of mathematics, was written. The inventions of analytic geometry, and later the calculus, were just around the corner. But the integration of mathematics into the physical world was well on its way before these innovations came along. With this episode, along with others, we might enrich our understanding of the history of mathematics by following a few paths that are now overgrown with weeds, but were once major thoroughfares.

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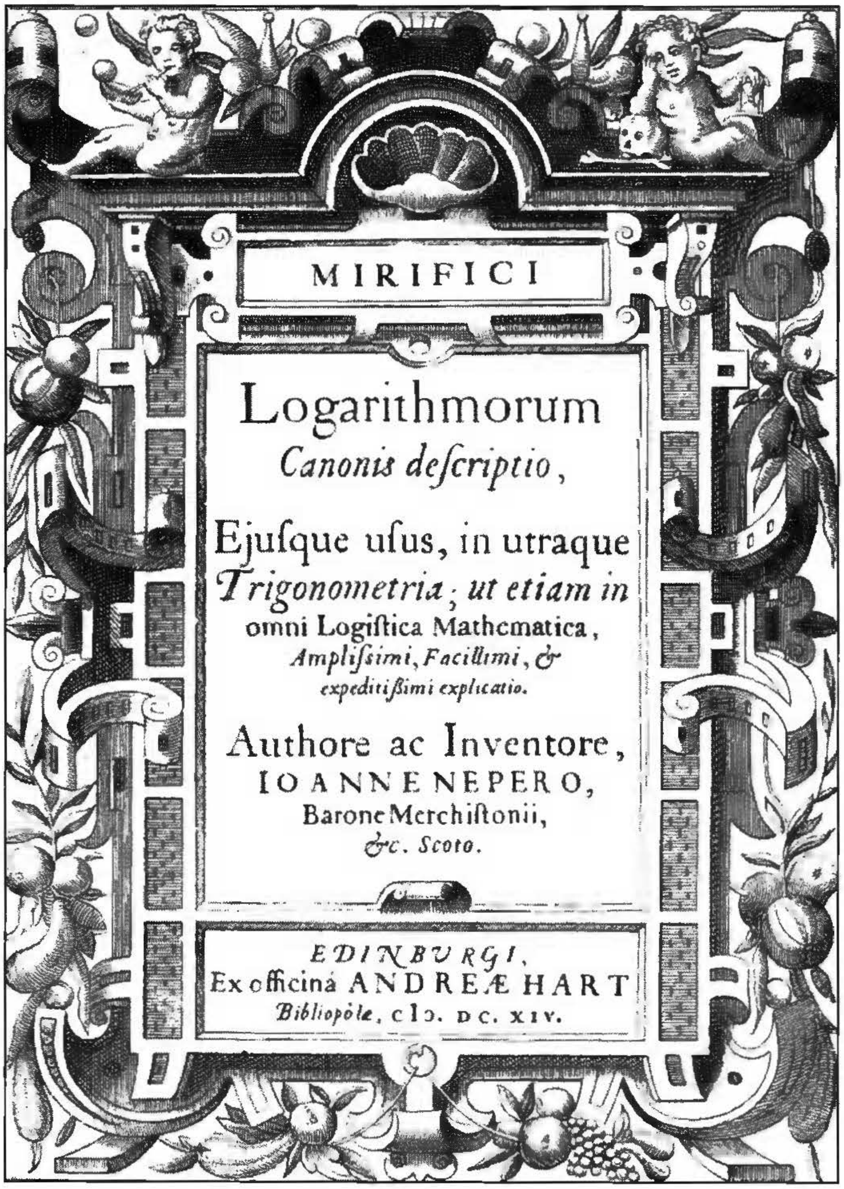
MIRIFICI
Logarithmorum
Canonis deſcriptio,
Ejuſque uſus, in utraque
Trigonometria; ut etiam in
omni Logiſtica Mathematica,
Ampliſsimi, Facillimi, &
expeditiſsimi explicatio.
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&c. Scoto.
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Figure 2. *Title page, John Napier,* Mirifici logarithmorum canonis description *(Edinburgh, 1614)*