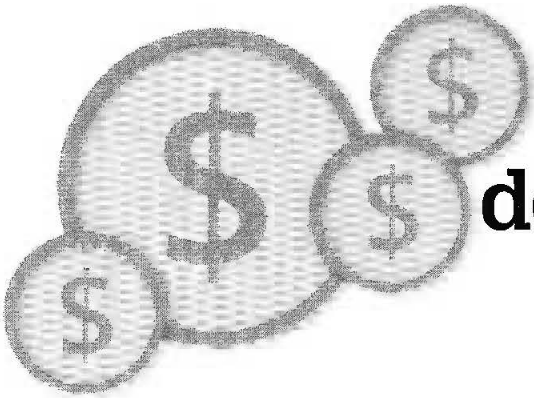
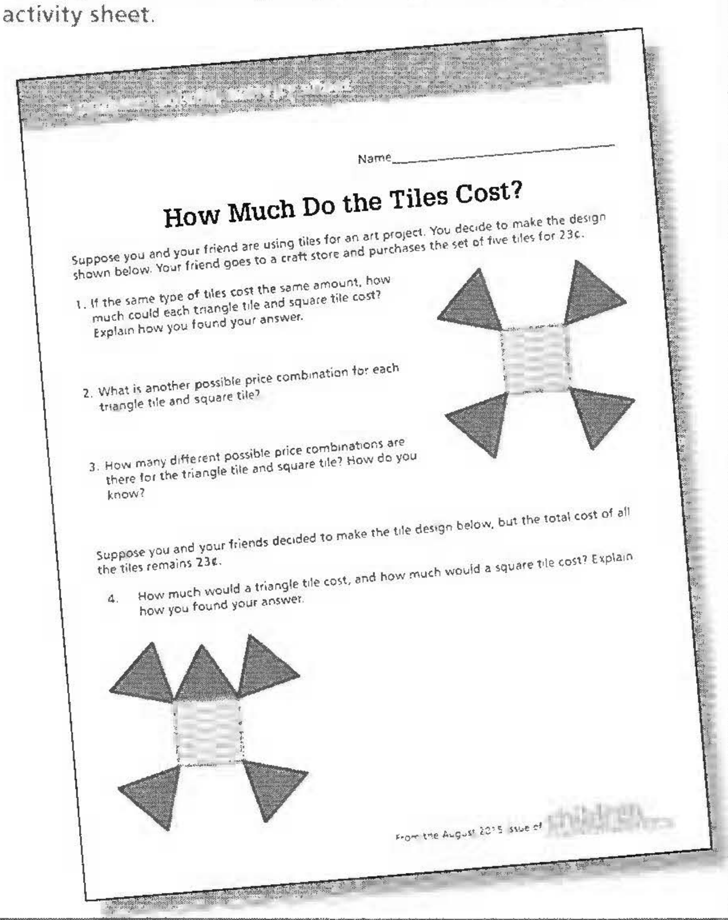
Sarah Quebec Fuentes



How much do the tiles cost?

The August 2015 problem scenario presents students with a design composed of square and triangular tiles as well as the total cost of the tiles needed to make the design. Students must determine the cost for one square tile and one triangular tile. Go to http//:www.nctm.org/tcm, All Issues to access the full-size activity sheet.



Name________

How Much Do the Tiles Cost?

Suppose you and your friend are using tiles for an art project. You decide to make the design shown below. Your friend goes to a craft store and purchases the set of five tiles for 23¢.

1. If the same type of tiles cost the same amount, how much could each triangle tile and square tile cost? Explain how you found your answer.

2. What is another possible price combination for each triangle tile and square tile?

3. How many different possible price combinations are there for the triangle tile and square tile? How do you know?

Suppose you and your friends decided to make the tile design below, but the total cost of all the tiles remains 23¢.

4. How much would a triangle tile cost, and how much would a square tile cost? Explain how you found your answer.

From the August 2015 issue of

Algebraic reasoning should be incorporated throughout the K–grade 12 curriculum. The How Much Do the Tiles Cost? problem is a high-level task that fosters algebraic reasoning and problem solving, is accessible to students across the elementary school grades, and can be solved in multiple ways. Rachael Quebec implemented the problem in her first-grade class at Joseph Estabrook Elementary School in Lexington, Massachusetts, providing a window into students' algebraic reasoning in the early elementary grades.

Introducing the problem

Quebec regularly uses a workshop model. While she works with a small group of students, the others freely move to and participate in various activities that preview, develop, reinforce, and extend the current focal mathematics learning goals. During the week before implementing the How Much Do the Tiles Cost? problem, Quebec previewed the task by presenting the 10¢ Designs activity to her class. Students received prices for various tiles and had to determine the total cost of several tile designs (see **fig. 1**). Because the total cost of each tile design was 10¢, as Quebec introduced the tiles task, she simultaneously reinforced the early number-sense ideas of part-part-whole and benchmarks of ten.

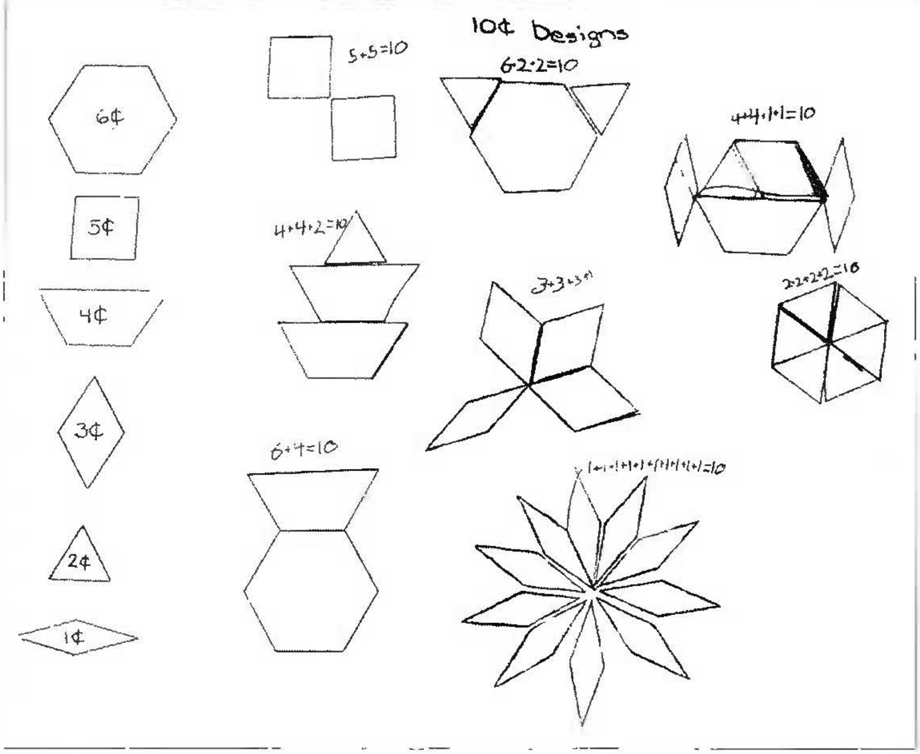
To launch the How Much Do the Tiles Cost? problem, Quebec met her first graders on the carpet and shared the tile design (from the activity sheet) with this story:

> Suppose you and your friend are using tiles for an art project. Your friend goes to a craft store and purchases the set of five tiles for 23¢. If the same type of tiles cost the same amount, how much could each triangle tile and square tile cost?

Quebec reminded her students about the 10¢ Designs activity and emphasized that instead of being given the tile prices and

FIGURE 1

During the previous week, this first-grade teacher had used the 10¢ Designs activity to reinforce the early number-sense concepts of part-part-whole and benchmarks of ten, simultaneously preparing her class for the How Much Do the Tiles Cost? task.



determining the total cost of a design, they were being given the total cost of the tile design and must determine the cost of individual tiles. Before dismissing students to work on the problem, Quebec reviewed her group-work expectations, such as listening actively, evaluating others' ideas, and explaining one's reasoning. Students returned to their table groups of four with a copy of the activity sheet to record their solution strategies.

Working on the problem

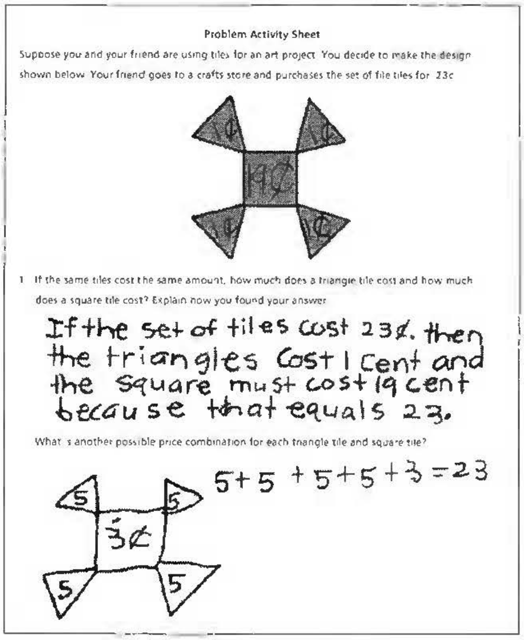
While groups tackled the task, Quebec walked around the classroom, monitoring students' strategies for solving the problem. To maintain the problem's high level of cognitive demand, she made sure not to tell students how to solve the problem. Instead, Quebec asked questions that built on the ways that students were already thinking about the problem. For example, one group equated the tile's number of sides with its cost (i.e., square tiles cost 4¢, and triangle tiles cost 3¢) and determined that the total cost of the design was 4 + 3 + 3 + 3 + 3 = 16¢. Quebec prompted these children to reread the problem and evaluate their answer. After the group realized that the total cost of the tiles must be 23¢, Quebec encouraged students to consider alternative strategies for solving the problem. Most groups initially generated one possible price combination totaling 23¢. Quebec circulated among the groups, asking questions to prompt students to find additional price combinations:

- Is there another possible combination of costs for the triangular and square tiles? Why or why not?
- How could you determine another combination of costs for the triangular and square tiles?

FIGURE 2

Student groups represented their solutions in various ways:

(a) Some groups labeled the tile design and wrote the corresponding number sentence.


Problem Activity Sheet

Suppose you and your friend are using tiles for an art project. You decide to make the design shown below. Your friend goes to a crafts store and purchases the set of tile tiles for 23¢

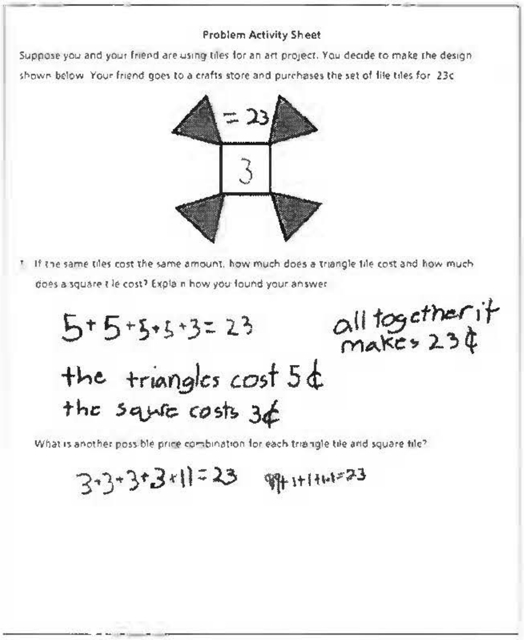
1 If the same tiles cost the same amount, how much does a triangle tile cost and how much does a square tile cost? Explain how you found your answer.

If the set of tiles cost 23¢. then the triangles Cost 1 Cent and the square must cost 19 cent because that equals 23.

What is another possible price combination for each triangle tile and square tile?

5+5 +5+5+3=23

(c) Some children simply wrote a number sentence only.


Problem Activity Sheet

Suppose you and your friend are using tiles for an art project. You decide to make the design shown below. Your friend goes to a crafts store and purchases the set of tile tiles for 23¢

1 If the same tiles cost the same amount, how much does a triangle tile cost and how much does a square tile cost? Explain how you found your answer.

5+5+5+5+3=23 all together it makes 23¢

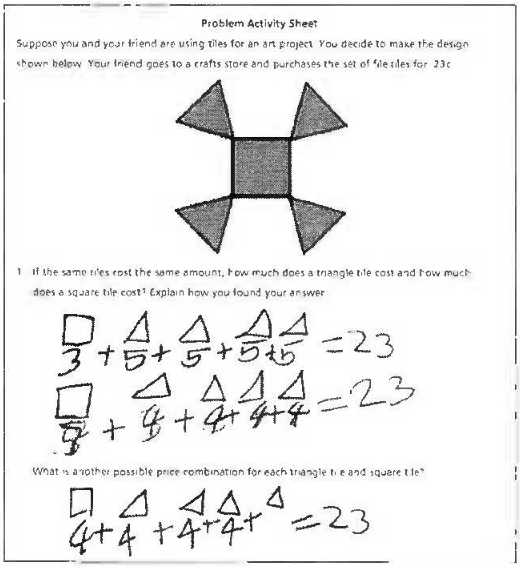
the triangles cost 5¢

the square costs 3¢

What is another possible price combination for each triangle tile and square tile?

3+3+3+3+11=23

(b) Other students marked each number in a number sentence with its corresponding shape.


Problem Activity Sheet

Suppose you and your friend are using tiles for an art project. You decide to make the design shown below. Your friend goes to a crafts store and purchases the set of tile tiles for 23¢

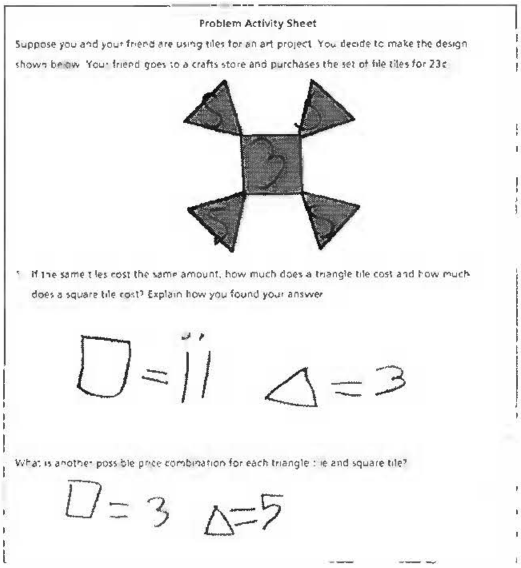
1 If the same tiles cost the same amount, how much does a triangle tile cost and how much does a square tile cost? Explain how you found your answer.

3+5+5+5+5=23

What is another possible price combination for each triangle tile and square tile?

4+4 +4+4+ =23

(d) These first graders equated the two shapes with their cost.


Problem Activity Sheet

Suppose you and your friend are using tiles for an art project. You decide to make the design shown below. Your friend goes to a crafts store and purchases the set of tile tiles for 23¢

1 If the same tiles cost the same amount, how much does a triangle tile cost and how much does a square tile cost? Explain how you found your answer.

□=11 △=3

What is another possible price combination for each triangle tile and square tile?

□=3 △=5

While monitoring her students at work, Quebec noted student approaches to the problem, their various price combinations, and representations of their solutions:

- Many groups assigned a price to the triangle tile, found the total cost of the four triangle tiles, and then counted on to 23 to determine the cost of the square tile.
- Each group found two or three of the five possible cost combinations.
- All groups found the price combination of 5¢ for a triangle tile and 3¢ for a square tile. Using benchmarks of five and ten, the students easily determined the total cost for the triangle tiles (5 + 5 + 5 + 5 = 20¢).
- None of the groups identified the price combination of 2¢ for a triangle tile and 15¢ for a square tile.

The groups represented their solutions in various ways: labeling the tile design and writing the corresponding number sentence (see **fig. 2a**), marking each number in a number sentence with its corresponding shape (see **fig. 2b**), writing a number sentence only (see **fig. 2c**), and equating the two shapes with their cost (see **fig. 2d**). Quebec used these observations to help structure the whole-class closing discussion about the problem.

Sharing and summarizing the problem

Students returned to the carpet, where Quebec asked them to share their problem-solving strategies and any patterns they noticed in the price combinations. One student conjectured that as the cost of the triangle increased, the cost of the square decreased. Quebec recorded this conjecture and collected students' various price combinations in a table on chart paper (see **fig. 3**). She introduced the table as a way to organize and represent the price combinations. As a class, students completed the table, starting with the last row for the 5¢ triangle tile price combination (i.e., the most commonly found combination) and adding the rows for the 4¢, 3¢, and 1¢ triangle tile price combinations. After realizing that the cost of the square tile for the 2¢ triangle tile was missing, students determined this cost and added it to the table. Referencing

FIGURE 3

Although none of the small groups had recorded the cost of the square tile when the triangle tile was 2¢, students noticed it was missing when the teacher recorded a table of possible price combinations during their whole-class discussion.

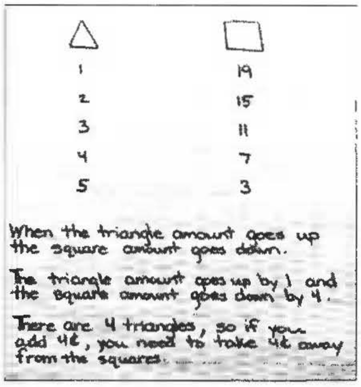
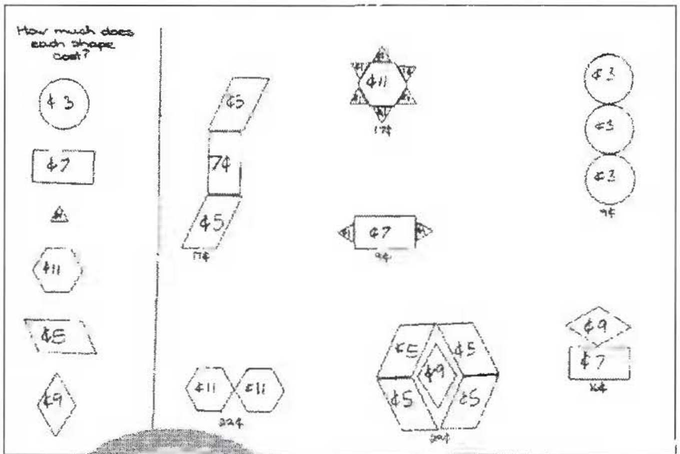
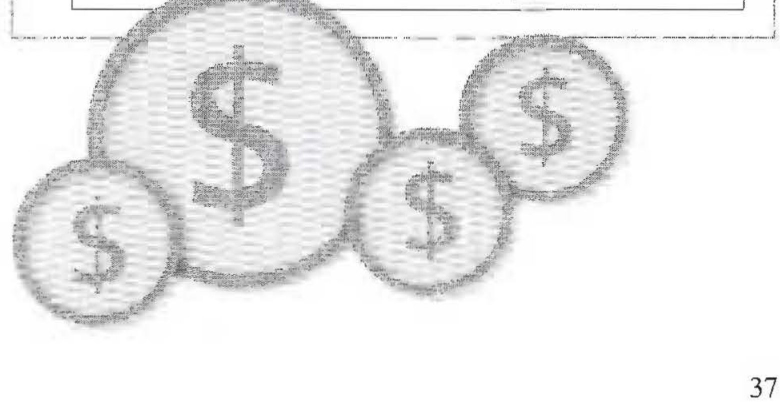


FIGURE 4

Subsequent to the How Much Do the Tiles Cost? task, Quebec incorporated the Shapes for Sale! activity into her math workshop. Using information they were given for tile design costs, students determined the cost of each tile shape.

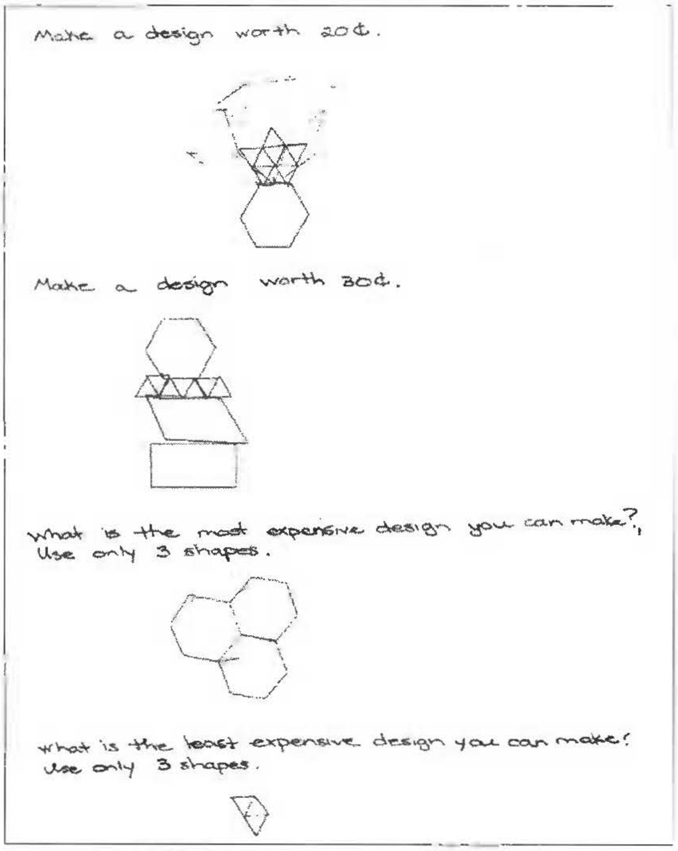


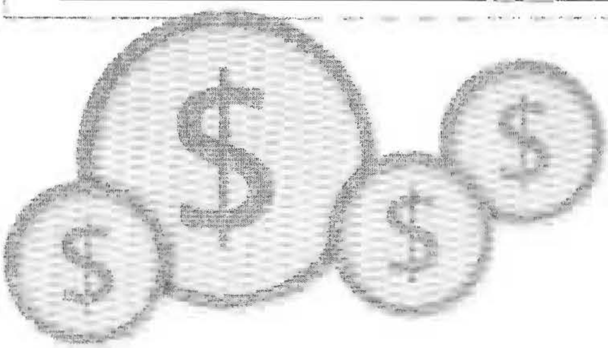


> "Students explicitly articulated and justified how a change in the cost of a triangle tile affects the cost of a square tile."

FIGURE 5

After students had determined the cost of each tile in the Shapes for Sale! activity, Quebec presented another task, Make a Design, during which students created designs to meet different criteria.





the table of all possible price combinations, students were able to explicitly articulate and justify how a change in the cost of a triangle tile affects the cost of a square tile (see the second and third observations in **fig. 3**).

Extending the problem

For the next several days, Quebec incorporated two related activities into her mathematics workshop. The first gave students the cost of various tile designs. Using this information, students determined the cost of each tile shape (see **fig. 4**). The second activity built on the first. On the basis of the determined costs of each tile, students created tile designs meeting different criteria (see **fig. 5**).

Quebec commented that the How Much Do the Tiles Cost? problem gave her students the chance to develop their algebraic reasoning through engaging with a problem that presented a realistic scenario, did not have an obvious and immediate answer, and had multiple solutions. Additional experiences with similar high-level tasks will give her students opportunities to enhance their ability to identify relationships between variables and expand their repertoire of representations.

Sarah Quebec Fuentes, s.quebec.fuentes@tcu.edu, is an associate professor of mathematics education at Texas Christian University (TCU) in Fort Worth, Texas. She is interested in the teaching and learning of mathematics for understanding. Her research projects focus on classroom discourse, teacher knowledge, mathematics curriculum materials, teacher self-efficacy, collaboration, and developing fraction sense. Edited by J. Matt Switzer, j.switzer@tcu.edu, an assistant professor of mathematics education at TCU. His research interests include preservice teacher education, early algebra, learning theory, learning trajectories, and identification of and interventions for students struggling in mathematics. Each month, this section of the department showcases classroom results of using problems presented in previous issues of *Teaching Children Mathematics*. Find detailed submission guidelines for all departments at **http://www.nctm.org/WriteForTCM**.