What Problem Are They Posing? Viewing Group Problem Solving Through an Enactivist Lens

Nat Banting

Instructional practices that facilitate deep mathematical understanding and elicit student thinking were thrust into the limelight by the National Council of Teachers of Mathematics (NCTM) by two and a half decades' worth of influential curriculum documents (1989, 2000, 2014). Closer to home, the Western and Northern Canadian Protocol (2008) called for similar pedagogical approaches in the mathematics classroom, including the incorporation of meaningful student discussion as a channel for developing mathematical understanding. Group problem solving presents itself as a key structure for the classroom teacher attempting to fulfill these curricular mandates. For this reason, group work in the mathematics classroom warrants examination from various theoretical frameworks, each of which brings different implications for employing the structure in the mathematics classroom. In this article, group problem solving is viewed through an enactivist lens, which stresses the evolutionary nature of problem solving as the learners and the problem mutually define one another. In other words, a problem does not live outside of the solvers; it is the interaction between knowers and task that shapes the nature of the problem. A mathematical problem, then, takes on a plural character as the solution to a provided task may involve several instances of problem drift, where a new problem becomes the focus of attention because it emerges as a relevant inquiry in the course of action with the evolving mathematical environment. A portrait of group action is provided to illustrate the evolutionary process of coming to know, and the implications of problem drift for teacher action are discussed.

Problem Solving for the Social Constructivist

This shift in the interpretation of a problem-solving activity is better explained when compared to the learning theory through which group problem solving is usually supported: social constructivism. Though there are many branches of social constructivism (Ernest 2010), I use the work of Lev Vygotzky (1978) to structure the discussion because of the importance he placed on language development and his construct of the zone of proximal development (ZPD) as a vehicle for its development, both of which are often discussed in introductory psychology courses and as part of teacher training. According to this theory, learning occurs when students internalize subjective meanings that fit into their social (and mathematical) worlds. In order to pull learning forward, interactions must fall within the learners' ZPD, a theoretical space that consists of tasks a novice could not carry out without an expert's guidance. The process of intentionally supporting learners to go beyond what they could do individually is called scaffolding. Small-group cooperative work, then, activates students as scaffolds. In other words, it allows students to encounter those with more sophisticated knowledge and thus provides scaffolding toward an understanding of the mathematical problem. It is the job of the students, through the use of socially negotiated language, to fit their understanding to invariants in the environment. It is the job of the mathematics teacher to provide a mathematical environment, in the form of a task to be completed, and to mitigate the learners' personal meaning of that environment. Teaching, then, is the process of diagnosing where in the solution process the learners are and deciding what assistance will allow them to move forward.

Problem Solving for the Enactivist

Enactivism and social constructivism are not disjoint (Reid 1996); both learning theories treat social interaction as the impetus for learning. However, a key difference is that constructivism is concerned with issues of fit, whereby learners use experience in an environment to construct sense from it. Enactivism, on the other hand, is "not so much about the invariants within the environment, but about the coordination of the knower and the environment" (Proulx and Simmt 2013, 66). According to enactivist theory, environment and cognition evolve alongside one another. In the case of group problem solving in the mathematics class, the environment consists of both the given task and the other learners working with the task. The actions of participants negotiate their way through a world that is not fixed or pregiven, but rather one "that is continually shaped by the types of actions in which we engage" (Varela, Thompson and Rosch 1991). The nature of the mathematical task that the learners are operating with and the mathematical knowledge used to arrive at coherence are constantly influencing each other. Problem solving is thus a process of "dynamic co-emergence of knowing agent-and-known world" (Davis 1995, 8). Acting within a mathematical environment changes the nature of the environment. This allows the characteristics of the task to become problematic for the learners, and, thus, the environment to trigger further action.

Problem solvers enter a mathematical environment with patterns of action established through their past interactions with environments. In other words, learners bring their history of viable, mathematical action with them when they begin to solve a problem. Interaction with the new environment begins here. The feedback provided by interacting with the structure of the new environment shifts the nature of the environment. Because the group now has more information about the nature of the task, there is further impetus for action, which is reciprocated by a shifting of the mathematical task. This is the image of evolutionary coming to know, the process of bringing forth a world of mathematical significance (Kieren and Simmt 2009; Proulx and Simmt 2013) where problem and solvers concurrently define one another. The task is not fixed; it is defined by the solvers. The actions of the solvers are not fixed; they are proscribed by the task. Problem solving, then, involves a fundamental circularity between knowers and their environment as they mutually specify one another (Davis 1996). According to enactivist theory, learners are not appropriating an individual, subjective meaning through the mechanism of social interaction, as proponents of social constructivism would contend; rather, through their interaction with others and environment, learners are bringing forth—enacting—mathematical meaning together.

In order to solve problems, problem solvers have the ability to pose "relevant issues that need to be addressed at each moment. These issues are not pregiven, but are enacted from a background of action" (Varela, Thompson and Rosch 1991, 145). In other words, the mathematical problem does not reside in the task itself; the structure of the task only triggers action. Through an enactivist lens, "prompts are given, not problems. Problems become problems when knowers engage with them, when they pose them as problems to solve" (Proulx and Simmt 2013, 70). That is, the prompt is not thought to contain the mathematics to be internalized. Instead, mathematical knowledge "emerge[s] from the knower's interaction with the prompt, through posing what is relevant in the moment" (Proulx and Simmt 2013, 69). Problem solving, then, is the process of coordinating intelligent action in an ever-changing process of problem posing. In order to do this, learners pose the problem that they believe will focus their mathematical action toward a coordination with the task's requirements. These relevant problems evolve as the group acts with the task, and I have termed this shift in the relevant problem that focuses students' action *problem drift*. Viewed through the lens of enactivism, learning is the process whereby problem solvers redefine their action in relation to the shifts in the mathematical environment; it is the process of bringing forth mathematical significance. Teaching, then, is the process of providing information, orienting attention and coordinating the possible in the mathematical environment (Towers and Proulx 2013). It requires attuning to the problem of relevance and coordinating further encounters with the environment to continue the problem-solving (posing) process.

A Classroom Portrait of Problem Drift

To illustrate the evolutionary process of knowledge and the notion of problem drift, an excerpt from a group of Grade 11 students working with the Squares Task (Appendix A) is analyzed. The Squares Task was a two-part task that asked Piper, Ben and Carter to first count the total number of squares that did not contain a blacked-out portion on an eight-by-eight grid and to determine the position of the blacked-out portion that would maximize the number of such possible squares. The excerpt begins as the group addressed the second question of the task. The use of an ellipsis represents a gap in transcription; all names are pseudonyms.

Piper: Wouldn't the smartest thing to do be to put it in a corner so you can get the largest amount of area?

Ben: Yeah, because then you'll get the. . . .

Carter: But we aren't talking about the biggest squares, we're talking about the most squares.

. . .

Piper: Regardless, there's going to be 55, one-by-one squares.

Carter: Yeah. Some things can't change.

Ben: OK, well like, the most that we got was two-by-two, right? Do you think if we moved the square anywhere on the piece it would change the amount of two-by-twos?

Carter: Two-by-twos wouldn't change.

Piper: Three-by-threes wouldn't change either.

Carter: Because it still takes the same amount of area.

. . .

Ben: If we just blanked out a corner, what would change?

Carter: Two-by-twos would be the same because you're still having the same area.

Piper: Yup.

Carter: Three-by-three is going to be the same, but you do get four-by-fours and five-by-fives.

Piper: Exactly, so the corner is the best!

This interaction between Piper, Ben and Carter maps an example of problem drift as their action couples with the environment. Not included in the episode, for sake of brevity, is the solution strategy for question one of the squares task (see Appendix A). The group used a combination of multiplication and subtraction operations to count the number of one-by-one squares and then decided to use a system of dots (Figure 1) to count the two-by-twos. After all, the prompt asked them how many, and the counting strategy paired well.

As the group began to address question two of the squares task, there was a converging of past histories. Piper suggested that moving the square as far out of the way as possible would be most effective, and Carter was quick to remind her that the goal of the task was not to create the most space, but to create the most squares; this action revealed hesitancy toward using the mathematical idea of area to solve the problem. For Piper, the relevant problem might be framed as, "What is the largest possible square we could fit on the grid?", but Carter's action signalled that the relevant problem for him was, "How can we create the largest number of squares?" The structure of the problem offered both of these possibilities, but Piper and Carter's mathematical pasts led them in different initial directions. A social constructivist might interpret Piper and Carter as having constructed different understandings of the nature of the task, but an enactivist sees the onset of two very different worlds of mathematical significance—one interacting with concepts of size and area, and the other with systematized enumeration.

I, as the provider of the task, anticipated the relevant problem to become, "Where do you place the blacked-out portion to create the most squares?", but Piper's comment that the number of one-by-one squares would not change regardless of the placement caused the group to attend to a new problem that they deemed important to the resolution of the task; namely, "Which sizes of squares will be unaffected by movement of the blacked-out portion?" In other words, Piper's comments produced problem drift. This was immediately validated by the other group members. Ben extended this line of action by wondering if Piper's pattern would remain true for the two-by-two squares. At the end of this episode, the action of all three students centred on the problem that was deemed as relevant to move toward a solution. The group was not solving the problem I encountered ("Where do you place the blacked-out portion to create the most squares?"); rather, they were coordinating their actions to solve the new, drifted problem, "What is the largest possible square we can create that doesn't include the blacked-out portion?" This was something quite similar to the proposition made by Piper at the onset of action, but quite different than the counting of squares that they engaged in throughout the middle portion of their time with the task—to verify that the number of squares was unaffected by the movement of the blacked-out section. For the enactivist, the mathematics emerged through interaction with the prompt, and the prompt was reciprocally redefined by the group's action on it. The process of problem solving (posing) evolved within and alongside the mathematical environment. Piper, Ben and Carter all became convinced of the conjecture that if larger squares are possible, then more squares are possible, and their solution can be seen shaded in the bottom left corner of the grid in Figure 1.

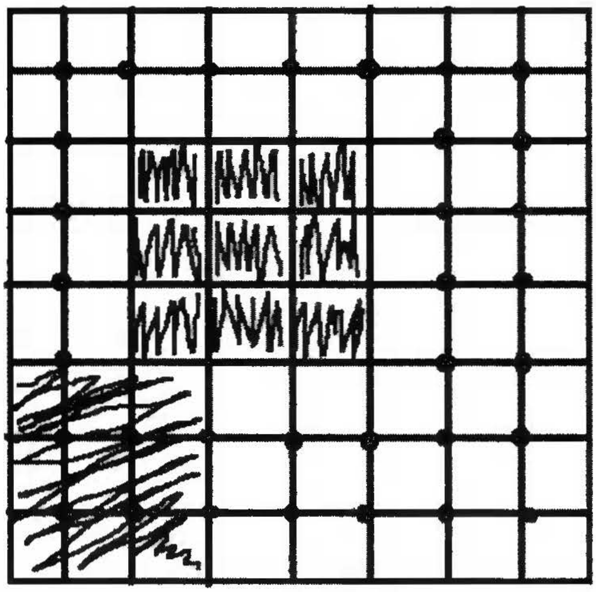


Figure 1. *The group's final workspace. When counting the two-by-two squares, each dot represented the centre of a two-by-two square. Their final solution was shaded in the bottom-left corner.*

The task afforded many possible actions; counting squares presented itself as viable for question one. Question two triggered the group toward a much different collective understanding, one where the notion of area emerged as crucial to the task. At the end of the episode, the group arrived at the understanding that maximizing the number of possible squares is the same as maximizing the area of the largest possible square. Their action led them to play with the notions of area and quantity in a geometric and arithmetic amalgam—a much different world of mathematical significance than was originally available to all three.

Implications for Teaching

How, then, should teachers act when viewing group problem solving through the lens of enactivism? The recognition of problem drift affects how teachers choose, prepare and enact problem solving (posing) activities in the classroom. First, the understanding that mathematics is an actively evolving phenomenon implies that classroom structure should provide spaces for student action. Problem-solving tasks should be designed to make students active producers (rather than strictly consumers) of mathematics, and provide arenas for worlds of significance to manifest. Teachers are called to include tasks with multiple avenues for conceptualization (like the arithmetic and geometric notions emerging through the Squares Task) and allow for student choice, conjecture, discussion, disagreement, justification and refutation. These visible (and audible) mathematical interchanges widen the sphere of mathematical action.

Second, problem drift requires teachers to be intentional in how they anticipate student responses to problem-solving (posing) activities. The goal is not to engineer a situation where students will act in particular ways; rather, it is to anticipate which features of the task will grab students' attention and to decide what intervention may trigger further student action. However, no matter how much teachers anticipate student interactions with the mathematical environment, the evolutionary character of knowing resists predictability. It is crucial that teachers, as they become fully involved in the knowing action in their classroom, remain flexible when student action breaches the sphere of anticipated strategy. To allow room for problem drift, lesson planning needs to give way to lesson preparing.

Third, while enacting problem-solving tasks, teachers must attune themselves to problem drift—what the group has posed as a suitable way forward. Attuning to a group's mathematical knowing begins with the identification of problem drift in the moments of teaching. This is more than the recognition that groups may find several solution pathways through a task or hold different constructions of the problem—it is the recognition that groups may actually be solving several different problems altogether. It means that the orienting question of teaching moves away from "How did they solve the problem?" and toward "What problem are they posing?" Viewing group problem solving (posing) through an enactivist lens allows the teacher to see the mathematical as plural, as emerging in the moment. We can no longer speak of pathways within a single problem, but must rather think of each problem as unique, emerging through the group's action with the task. The teacher is then tasked with anticipating possibilities, recognizing where the problem has drifted in classroom activity and acting with the groups to further the mathematical action.

References

Davis, B. 1995. "Why Teach Mathematics? Mathematics Education and Enactivist Theory." *For the Learning of Mathematics* 15, no 2: 2–9.

———. 1996. *Teaching Mathematics: Toward a Sound Alternative.* New York: Garland.

Ernest, P. 2010. "Reflections on Theories of Learning." In *Theories of Mathematics Education: Seeking New Frontiers*, ed B Sriraman and L English, 39–47. New York: Springer.

Kieren, T, and E Simmt. 2009. "Brought Forth in Bringing Forth: The Inter-Actions and Products of a Collective Learning System." *Complicity: An International Journal of Complexity and Education* 6, no 2: 20–28.

National Council of Teachers of Mathematics (NCTM). 1989. *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va: NCTM.

———. 2000. *Principles and Standards for School Mathematics.* Reston, Va: Author.

NCTM. 2014. *Principles to Actions: Ensuring Mathematical Success for All.* Reston, Va: NCTM.

Proulx, J, and E Simmt. 2013. "Enactivism in Mathematics Education: Moving Toward a Re-Conceptualization of Learning and Knowledge." *Education Sciences and Society* 4, no 1: 59–79.

Reid, D A. 1996. "Enactivism as a Methodology." In *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education*, ed L Puig and A Gutiérrez, 203–09. Valencia, Spain: Universitat de Valéncia.

Towers, J, and J Proulx. 2013. "An Enactivist Perspective on Teaching Mathematics Reconceptualizing and Expanding Teaching Actions." *Mathematics Teacher Education and Development* 15, no 1: 5–28.

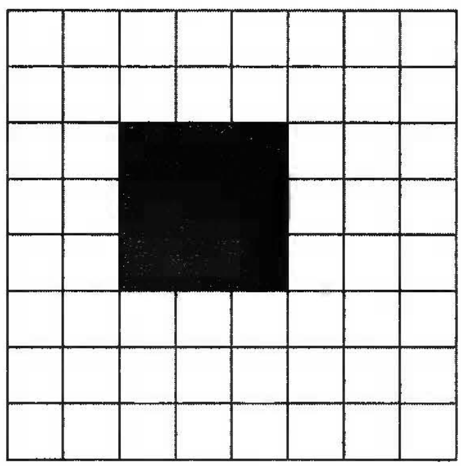
Varela, F J, E Thompson and E Rosch. 1991. *The Embodied Mind: Cognitive Science and Human Experience.* Cambridge, Mass: MIT.

Vygotzky, L S. 1978. *Mind in Society.* Ed M Cole, V John-Steiner, S Scribner and E Souberman. Cambridge, Mass: Harvard University Press.

Western and Northern Canadian Protocol. 2008. *The Common Curriculum Framework for Grades 10–12 Mathematics.* www.wncp.ca/media/38771/math10to12.pdf (accessed January 5, 2017).

Nat Banting is a high school mathematics teacher in Saskatoon, Saskatchewan, and a recent MEd graduate from the University of Alberta. He blogs his thoughts concerning the teaching and learning of mathematics at musingmathematically.blogspot.ca.

Appendix A



The Squares Task

1. How many squares (of any size) can be drawn on the above eight-by-eight grid that do not include any area from the blacked-out portion?
2. Where would you place the blacked-out portion to maximize the number of possible squares?