delta-k

JOURNAL OF THE MATHEMATICS COUNCIL OF THE ALBERTA TEACHERS' ASSOCIATION

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Volume 48, Number 2 June 2011

39 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 123456789 12345678

Special Issue:
Early Childhood Mathematics

Guidelines for Manuscripts

delta-K is a professional journal for mathematics teachers in Alberta. It is published twice a year to

- promote the professional development of mathematics educators, and
- stimulate thinking, explore new ideas, and offer various viewpoints.

Submissions are requested that have a classroom as well as a scholarly focus. They may include

- personal explorations of significant classroom experiences;
- descriptions of innovative classroom and school practices;
- reviews or evaluations of instructional and curricular methods, programs or materials;
- discussions of trends, issues or policies;
- a specific focus on technology in the classroom; or
- a focus on the curriculum, professional and assessment standards of the NCTM.

Suggestions for Writers

1. *delta-K* is a refereed journal. Manuscripts submitted to *delta-K* should be original material. Articles currently under consideration by other journals will not be reviewed.
2. If a manuscript is accepted for publication, its author(s) will agree to transfer copyright to the Mathematics Council of the Alberta Teachers' Association for the republication, representation and distribution of the original and derivative material.
3. All manuscripts should be typewritten, double-spaced and properly referenced. All pages should be numbered.
4. The author's name and full address should be provided on a separate page. If an article has more than one author, the contact author must be clearly identified. Authors should avoid all other references that may reveal their identities to the reviewers.
5. All manuscripts should be submitted electronically, using Microsoft Word format.
6. Pictures or illustrations should be clearly labelled and placed where you want them to appear in the article. A caption and photo credit should accompany each photograph.
7. References should be formatted consistently using *The Chicago Manual of Style*'s author-date system or *The American Psychological Association (APA)* style manual.
8. If any student sample work is included, please provide a release letter from the student's parent/guardian allowing publication in the journal.
9. Articles are normally 8–10 pages in length.
10. Letters to the editor or reviews of curriculum materials are welcome.
11. Send manuscripts and inquiries to the editor: Gladys Sterenberg, 195 Sheep River Cove, Okotoks, AB T1S 2L4; e-mail gladyss@ualberta.ca.

MCATA Mission Statement

Providing leadership to encourage the continuing enhancement of teaching, learning and understanding mathematics.

delta-K

Volume 48, Number 2 June 2011

Special Issue: Early Childhood Mathematics

CONTENTS

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From the Editor's Desk

Gladys Sterenberg

Summer! A time of renewal, recovery and rest. A time of warmth and sunshine. A time of vacation and leisure. A time of planning, regrouping, reconsidering and reading. As I look forward to gardening, hiking and camping, I am reminded of the importance of having time to reflect on my past year of teaching and to envision the upcoming year. I hope you will find the articles in this issue a source of inspiration.

This issue of *delta-K* is a special focus issue on early childhood mathematics. Lynn McGarvey, coeditor of this special issue, describes the importance of early mathematics learning in her editorial, which follows. This focus may seem strange, since we have spent the past year implementing program changes at the secondary level. However, I believe that all readers will benefit from reflecting on the importance of making mathematics meaningful and teaching for deep understanding. These articles provoke thought about how young children learn and how teachers build on these early childhood experiences. I believe the ideas presented in these articles can be adapted to elementary and secondary mathematics classrooms; certainly they contribute to a more holistic way of understanding our professional practice.

This issue showcases research and teaching ideas from authors across Canada. To begin, an investigation of children's images of mathematicians is presented by George Gadanidis. His research has direct implications for teachers of all students as we seek to challenge popular views of mathematicians and offer students a positive image of the work we engage in. Florence Glanfield and Shaun Murphy describe an example of how students and teachers are engaged in identity-making through mathematics assessment. Again, their description of differentiated assessment by engaging students in conversations through interviews can inform our teaching in classrooms across all grades.

As an example of what we can learn by listening to children's conversations, Lynn McGarvey presents a task that reveals students' reasoning about patterns. She challenges our notions of developmental stages of learning and provokes us to consider how we might support children's learning and mathematical thinking. Kim Gravel also investigates children's mathematical learning through a process of pedagogical documentation. Her work involves toddlers' understanding of mathematics and emphasizes how children's learning can be made visible. Sylvia Malo uses pedagogical documentation to frame her journey of learning more about how children develop number sense through subitizing. The process of pedagogical documentation holds promise for learning within our professional contexts and can be applied in all classrooms.

Finally, the collection of photographs from the 2010 MCATA conference shows teachers engaged in mathematical and pedagogical learning. I invite you as mathematics teachers to become actively involved in our association. Attend the upcoming MCATA conference, nominate colleagues for the awards sponsored by the association, volunteer to be a member of the executive. Seek opportunities to build professional relationships. Information is available on our website, www.mathteachers.ab.ca.

As always, I encourage you to consider publishing your teaching and scholarly ideas in *delta-K*. The guidelines are listed on the inside of the front cover. I would be more than willing to assist you with this process.

Enjoy your summer!

Early Childhood Mathematics

Guest Editor: Lynn McGarvey

Most teachers, parents and community members are very aware of the importance of early experiences in literacy. Parents start reading to their children as babies, and most preschool and primary schools have home reading programs. Yet early mathematics learning does not receive the same attention, and less is known about what mathematical experiences are important. There are likely many reasons for the lack of attention, but of significance are the cultural perceptions that mathematics is difficult and that young children are not capable of mathematical thinking. Mathematics is seen as abstract, and teaching children anything other than rote counting before they enter school is often deemed unnecessary and possibly even inappropriate. Recently, however, research and media have emphasized the importance of early mathematics to address a variety of concerns, such as ensuring future school success, closing knowledge gaps based on socioeconomic status and contributing to the global scientific society. While these concerns may be politically prestigious, perhaps the most important reason for attending to early mathematics is simply the recognition that young children are capable of significant mathematical thinking and learning. Providing opportunities for children to learn and do mathematics is not just to ensure their future success in school, close any perceived achievement gap or help them get good jobs decades later in life—mathematics, just like language and literacy, is a way of thinking, sense-making, describing and participating in the world. Giving children opportunities to engage in sufficiently challenging mathematics allows them to experience, participate in and make sense of their present-day environments.

Early mathematics teaching is not the same in each classroom or from lesson to lesson. There are no rules or prescriptions for teaching. It need not be student centred or teacher directed. However, excellence in teaching draws on and extends children's knowledge and interests, helps children develop a vocabulary of mathematics, and allows children to make conjectures, formulate problems, and engage deeply in mathematical questions, problems and ideas.

For this special issue of *delta-K* we invited manuscripts on teaching and learning mathematics with young children (pre-K to Grade 3). We asked for papers that provided classroom-tested activities and teaching strategies, offered insight into children's thinking and problem-solving strategies, addressed challenging classroom issues and shared findings from classroom-based research. In the articles selected for this issue we see teachers and researchers engaging with young children in a variety of ways that demonstrate children's capabilities, interest and understanding of significant mathematics.

Meet Your Executive

Back Row (L–R): Marj Farris, Olive Chapman, Gladys Sterenberg, Rod Lowry, Daryl Chichak, Donna Chanasyk, David Martin, Christine Henzel, Lisa Everitt

Middle Row (L–R): Shauna Rebus, Jennifer Baerg, Carmen Wasylynuik, Indy Lagu, Tancy Lazar

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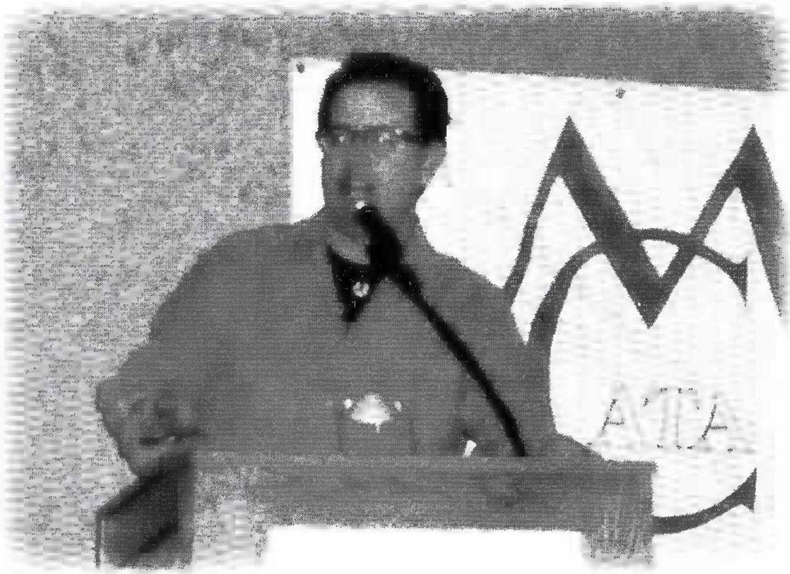
Rod Lowry,
conference committee cochair



David Martin, conference committee cochair

Photographic Memories: 2010 MCATA Conference

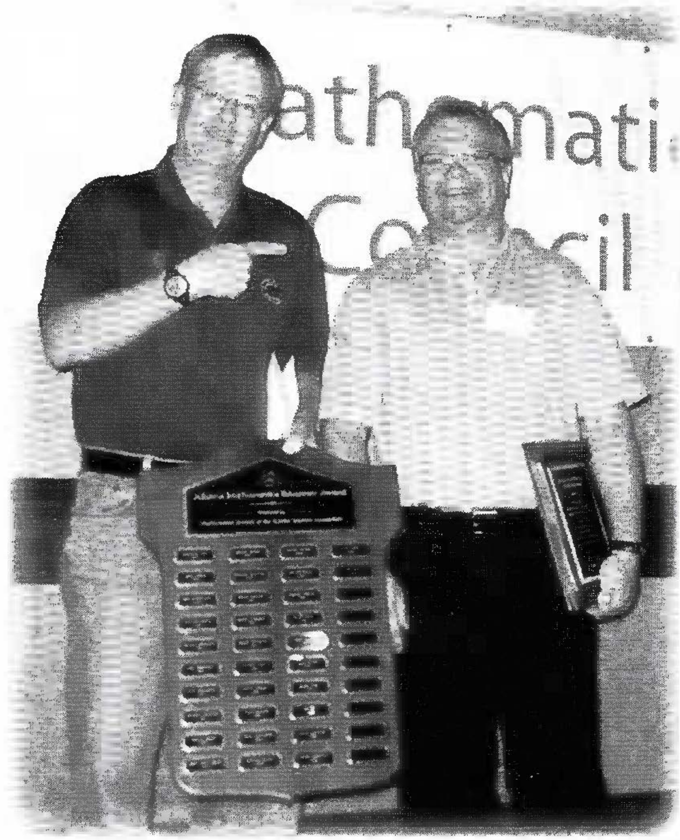
Keynote Speakers

Olive Chapman

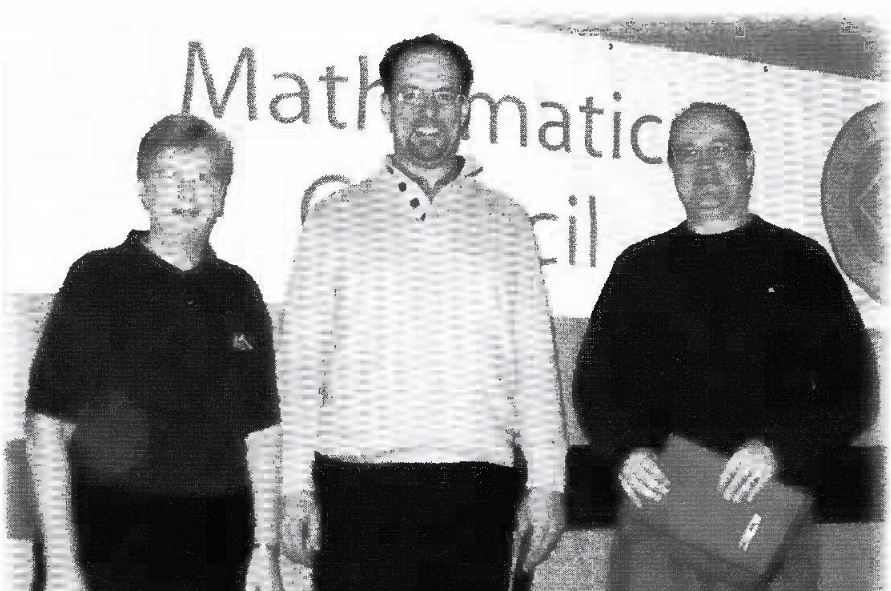
Arthur T Benjamin

Awards

Art Jorgensen Award Recipient: Karen Viersen (presented by Carmen Wasylynuik)



Alberta Mathematics Educator Award Recipient: Ed Tobin (presented by Keith Van De Keere)



Friends of MCATA: Mark Mercer and John Scammell (presented by Donna Chanasyk)

Conference Fun

Displays

Door prizes

Participants

Fall Symposium

What Does a Mathematician Do?

George Gadanidis

What do young students know about mathematicians, and how do we find out what they know? Picker and Berry (2001, 202) suggest that finding out more about students' images of mathematics can help teachers "understand their attitudes toward, misconceptions about, and opinions of the subject." They also suggest that "one way to discover these attitudes is to ask your students to create a drawing of a mathematician." We followed Picker and Berry's advice and asked 18 students in a Grade 2 class and 17 students in a Grade 3 class to draw mathematicians at work.

Picker and Berry (2000) conducted an international study a decade ago, and asked 476 students (age 12–13) to draw a mathematician at work. The major finding of the study was that "mathematicians are essentially invisible, with the result that pupils appear to rely on stereotypical images from media to provide image of mathematicians when asked" (p 88). Seven themes emerged from the drawings made by students in different countries (p 74):

(1) *Mathematics as coercion*—students "drew mathematicians as teachers who use intimidation, violence, or threats of violence to make their pupils learn material. This was a completely unexpected theme that emerged from the drawings";
(2) *The foolish mathematician*—"mathematicians were depicted as lacking common sense, fashion sense, or computational abilities";
(3) *The overwrought mathematician*—"mathematicians were depicted as looking wild and being overstrained";
(4) *Mathematicians who can't teach*—"a classroom is drawn which the mathematician cannot control, or in which he doesn't know the material";
(5) *Disparagement of mathematicians*—mathematicians "as being too clever or in some other way contemptible";
(6) *The Einstein effect*—drawings with a reference to Albert Einstein. Usually, those images were influenced by media, including books and cartoons;
(7) *Mathematicians with special powers*—including wizardry and math potions. "Something extraordinary is necessary in order to do mathematics."

Picker and Berry (2001) repeated the draw-a-mathematician survey with 201 ethnically diverse Grade 7 students in two schools and found that "no drawings emerged that represented that diversity" (p 204). Most drawings showed middle-aged males with glasses and/or a beard, bald or with weird hair, at the blackboard or computer. Such images raise important issues about how popular culture may deter many people from enjoying and studying math and may create stereotypes of mathematicians as mainly white, middle-class men; the stereotypes, in turn, could discourage other groups from engaging in math (Economic and Social Research Council 2008).

Grade 2 and 3 Students' Views of Mathematicians

We engaged the 35 Grade 2 and 3 students with the same tasks used by Picker and Berry in their study:

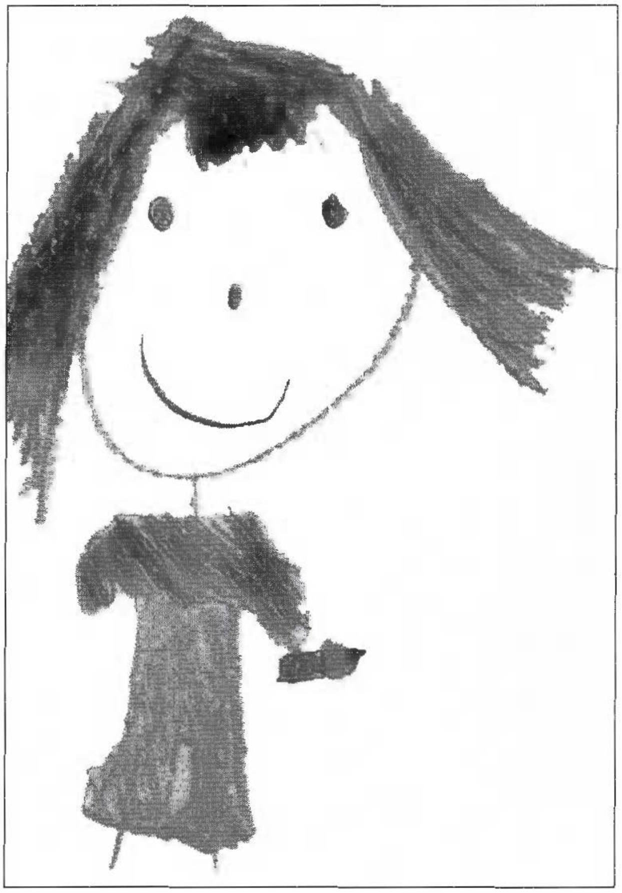
(1) When would somebody need to hire a mathematician?
(2) Draw a mathematician at work; and
(3) Explain your drawing in writing.

It should be noted that these two classes were part of a research project looking at "big math ideas across the grades," and that the draw-a-mathematician activity was completed before the research team started working in these classrooms.

Figures 1 and 2 show two typical student drawings. Overall, the following patterns emerged:

(1) Most students (30 out of 35) associated the work of a mathematician with that of a teacher or tutor. This parallels the Picker and Berry finding that

Figure 1. Grade 2 student: "It is a pensell [pencil], because he is helping me with my math."



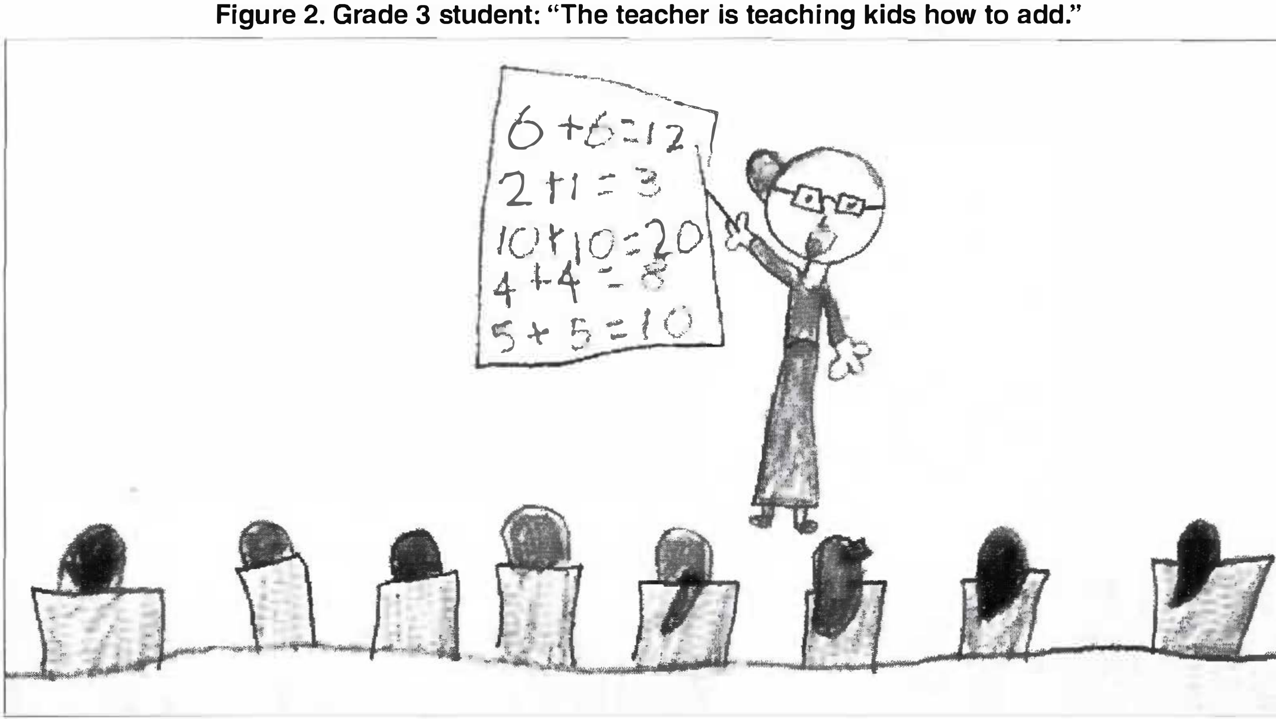
students do not have a good sense of what a mathematician does. Some students (5 out of 35) did state that a mathematician is someone who does math or works on math.

(2) Unlike the Picker and Berry study, in which students typically depicted mathematicians as male, the Grade 2 and 3 students depicted female mathematicians in 7 out of 18 drawings where we could clearly discern gender from the drawing or the description. In the Grade 2 class, taught by a female teacher, six boys and one girl depicted a male mathematician, and four girls depicted a female mathematician. In the Grade 3 class, taught by a male teacher, one boy and three girls depicted a male mathematician and three girls depicted a female mathematician. Interestingly, all of the female mathematicians were depicted by female students.

(3) Unlike the Picker and Berry study, in which mathematicians were depicted in negative and sometimes threatening ways, the Grade 2 and 3 students typically depicted smiling mathematicians and students in helping situations. In drawings where we could clearly discern facial expressions, most boys and girls depicted either smiling faces (25 out of 29) or faces with little expression (4 out of 29).

Although we cannot draw strong conclusions based on data from only 35 students, their drawings and some of the contrasts to the Picker and Berry study

Figure 2. Grade 3 student: "The teacher is teaching kids how to add."



do draw our attention to what Grade 2 and 3 students might know about mathematicians and how they might view them. It might be interesting to do a draw-a-mathematician activity in your own classroom. Your students' images could be compared to those from our Grade 2 and 3 classes or to those from the Picker and Berry study. Identifying the mathematician stereotypes your students depict and what knowledge they lack would be a first step toward planning classroom activities that disrupt their images and help them develop a better understanding of mathematicians. One resource that might be useful is discussed below.

Windows into Mathematicians

How students see mathematicians also draws our attention to what teachers know about mathematicians and how we convey this knowledge to our students. To get better insights into the work and thinking of mathematicians, I have been interviewing mathematicians through a project funded by the Fields Institute, Windows into Elementary Mathematics, which invites prominent mathematicians to discuss topics from elementary mathematics.

In one of the interviews, Megumi Harada, from McMaster University (see Figure 3), who works in the area of geometry, talks about parallel lines and disrupts the idea that parallel lines never meet. We actually use these ideas from Harada's interview in Grades 1–3 classrooms to engage students with explorations of lines on the sphere on which we all live. You can see how a Grade 2 teacher explored this topic at www.edu.uwo.ca/mpc/bigideas/parallel, and you can hear her students singing a song based on their writing at www.edu.uwo.ca/mathscene/mathfest2009/mathfest232.html. Below is the sequence of activities in the classroom. The activities are designed to offer students opportunities to be surprised mathematically and connect emotionally with math ideas through characters in children's literature (Gadanidis, Hughes and Borba 2008).

- Students looked for parallel lines around the classroom (tiles on the floor and ceiling, lines on the cupboard doors, wires on the guinea pig cage and so forth).
- We considered the following puzzle and students guessed at the colour of the bear.
  - Molly steps out of her tent.
  - She walks south 1 kilometre.
  - She walks west 1 kilometre.

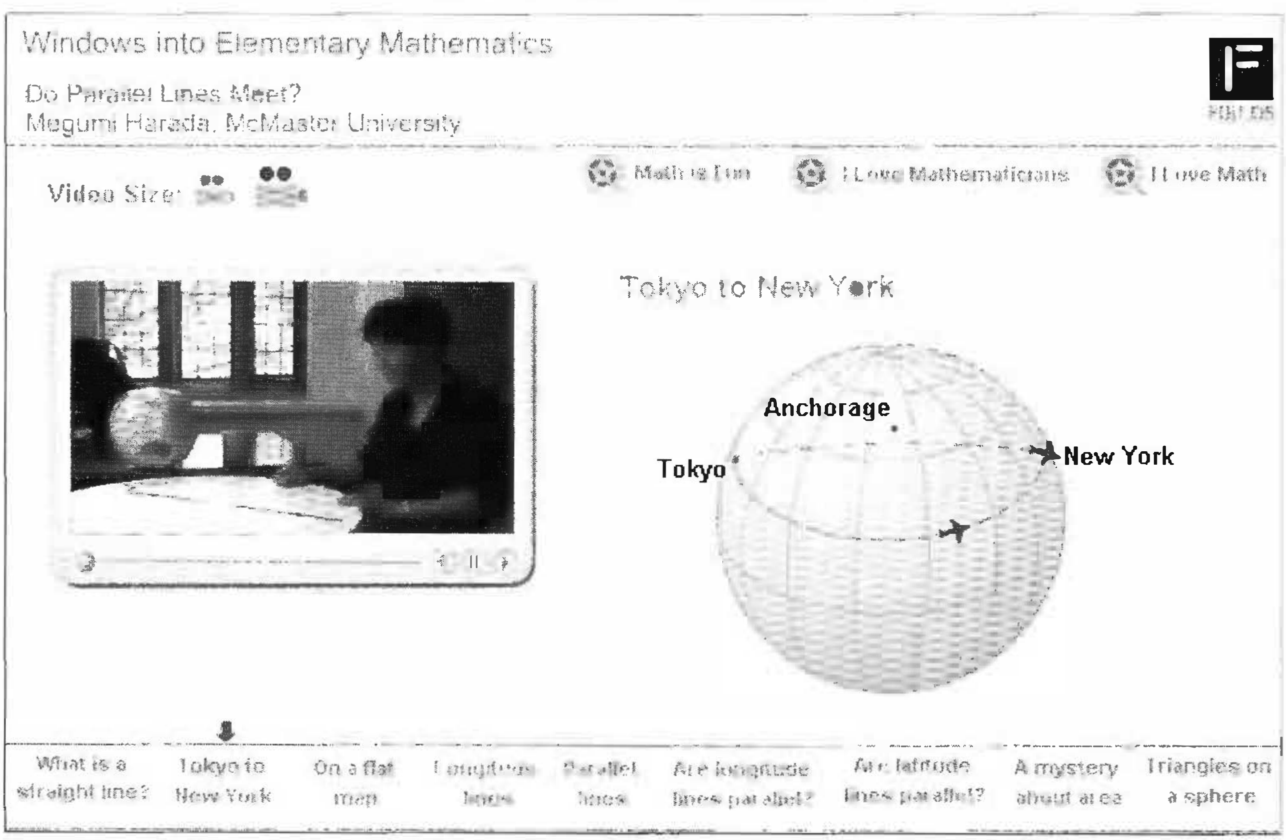


Figure 3. Megumi Harada interview.

    - She sees a bear, and gets scared.
    - She runs north 1 kilometre, arriving back at her tent.
    - How is this possible?
    - And what colour was the bear?
- We read the story *Do Parallel Lines Meet?* (Gadanidis and Gadanidis 2009).
- We explored the story and the puzzle by looking at lines on the globe.
- Students drew pictures and wrote to describe what they learned.
- Student writing was compiled to create the song shown in Figure 4.
- Students met another Grade 2 class in the library and they sang their song together.

In her interview, Megumi Harada also provides a unique insight into her attraction to mathematics:

> I love mathematicians. When I was in university I studied a lot of things. I studied literature, I studied anthropology, I studied linguistics, I studied philosophy. It wasn't until my fourth year of university that I decided to pursue math. So I was doing a lot of other things before that—in fact, I was an East Asian Studies major before I was a math major. I knew a whole lot of people as a young student and I can say without any doubt that the math students were the most fun to be around, and I think it's because, as a group, mathematicians love what they do more than many, many other groups of people I know.... Mathematicians are a group of people who love math more than they love themselves. Somehow math is this huge, beautiful world that we're just a part of, we're just playing in it, swimming in it, and sometimes we find wonderful jewels embedded in it. Somehow the world of math is bigger than us. Somehow there's a sense of humility that mathematicians share that really keeps us a tight-knit community, a supportive community I'd like to think, and makes it really, really fun to work with and talk with and explore with other people who share that same passion.

To illustrate what Megumi Harada says about mathematics and about her work, we have used her words to write a song called "I Love Mathematicians" (see Figure 5). You can see a performance of this song at http://joyofx.com/music/mst-song2.html. Through a second project funded by the Fields Institute, called Joy of X, we perform this song, as well as student songs that emerge from our work in elementary school classrooms, in math concerts for elementary schools.

Conclusion

For the most part, what mathematicians do and what they are like as people remain invisible in our society. At the same time, the image of mathematicians that our students (and we, as teachers) hold can affect how we see and value mathematics, so it's important that we help our students better understand mathematicians. The Windows into Elementary Mathematics resource could be one source of such knowledge.

Figure 4. "Parallel Lines" song

Parallel Lines

Paaaraaalleeell lines
Paaaraaalleeell lines

Tiles on the ceiling
Lines on the cupboard
Wires on the guinea pig cage

Paaaraaalleeell lines

Parallel lines
Never meet
But they meet, at the North Pole

Paaaraaalleeell lines

The world is a sphere
A 3-D solid
The world is not flat like a circle

Paaaraaallell lines

Molly in her tent
How did she get back
She saw a bear, what colour was it?

Paaaraaalleeell lines

Molly went south
Then went west
Then went north, how did she get back?

Paaaraaalleeell lines

Parallel lines
in a triangle
At the North Pole, is how she got back

Paaaraaalleeell lines

Paaaraaalleeell lines

Figure 5. "I Love Mathematicians" song

I Love Mathematicians

I think math is beautiful
The geometry I do
Is so intuitive
Something I can doodle

I studied many things
Literature, anthropology
Linguistics, philosophy
But I love mathematicians

They are the most fun
They love what they do
More than many many
Other people I know

La lala lala
La lala lala
I love math
I love mathematicians
La lala lala
La lala lala
I love math
I love mathematicians

We stay up with a problem
Work on it together
This sense of solidarity
Attracts me to math

I love doing math
Everyone contributing
Reminding each other
Why we do what we do

Math is a treasure trove
Playing, swimming
Finding jewels in it
Math is bigger than me

La lala lala
La lala lala
I love math
I love mathematicians
La lala lala
La lala lala
It's what keeps me in math

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Gadanidis, G, J Hughes and M Borba. 2008. "Students as Performance Mathematicians." *Mathematics Teaching in the Middle School* 14, no 3: 168–75.

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George Gadanidis is an associate professor in the Faculty of Education of the University of Western Ontario. His most recent project involves helping students develop performance skills for answering the question, "What did you do in math today?" He enjoys turning student thinking into songs that he and his band perform for K–8 schools, with funding from the Fields Institute. You can see some of their music videos at www.joyofx.com. George also heads the Math Performance Festival, www.mathfest.ca.

Possibilities for Understanding Children's Mathematics Knowledge

Florence Glanfield and M Shaun Murphy

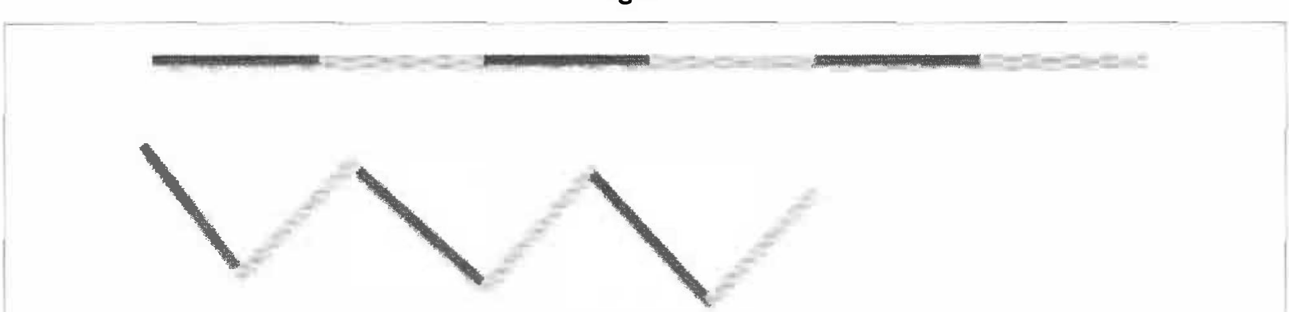
During the question about conservation of length, Michael, a child in Grade 1, was shown two rows of six rods laid end to end in a straight line. Michael was told that a bunny rabbit was hopping along each row, then he was asked if each rabbit had the same distance to go; Michael responded, "Yes." For the next question I altered one road by changing its path from straight to a zigzag pattern, using the same six rods (see Figure 1). Michael was again asked if the rabbits had the same distance to go and responded, "No."

I think Michael saw the two roads as a race—he stated his concern that the slight break between rods in the zigzag road might cause the rabbit to fall off the road, thus making the rabbit slower. He could not understand what *same length* meant. Length and equality were concepts I thought he understood from his work with length and distance months earlier in the classroom. He continually looked to me for some type of cue to help him understand what I was looking for in this question. I found it hard to explain what the question was asking without prompting him. I believe Michael thought I was trying to trick him. Often in my classroom I ask students how many ones are in the ones' house when there is zero or how many nickels are in a piggy bank that only has quarters and dimes. Because of this classroom trickery I feel Michael was looking for much more than the question asked (field note, February 10, 2009).

In this opening field note Michael and his teacher, Raina (the *I* in the field note), illustrated for us the complex understanding necessary when teachers and children are involved in assessment. Michael and Raina were in the process of demonstrating their knowledge, his of length or distance, and hers of mathematics education. This field note was a piece of an interview that was part of a three-year inquiry into children's and teachers' identity-making possibilities in mathematics assessment.[1] In the above field note both student and teacher wish to be understood as mathematical and knowledgeable, a common feature in any assessment.

The video of this assessment moment shows that Raina worked diligently to elicit a response from Michael that was correct with respect to what the assessment was focused on—the conservation of length. Michael, however, brought a number of contexts into play. He was concerned about the welfare of the rabbit, he saw the breaks between the rods as important and, in response to the question, "Did the rabbit travel the same distance?" he replied, "No." As our research team, which now includes Raina in a different role from that of assessor, watched and discussed the video, it was evident to us that the rabbit on the angled rods did not get as far on the surface of the desk as the one on the straight rods. A further complication, in Raina's thinking, was that Michael thought she was trying to trick him, which is an aspect of Raina's teaching that she uses to provoke the

Figure 1



children's thinking. Raina thought that Michael imagined that something else was at play in this assessment moment.

After video-recording Michael and Raina in the assessment, a member of our research team videoed Michael watching the assessment to see if he could comment about what he was doing; this was followed by an interview with Raina. In the interview, Raina raised concerns about the misunderstanding of distance and length. Raina knows Michael as a thinker. She knew that he was involved in an intellectual negotiation about length and distance, and she knew that he knew there would be a reason for the questions Raina was asking. Later, as our research team viewed both the assessment videos and the interview with Raina, we discussed what might have been happening in the assessment moment. In this space for thinking we began to see the complexity in understanding distance and length.

Distance and length, while seemingly similar, ask us to attend to different conceptions. Distance is defined as "the length of the line segment joining two points" (James and James 1992, 130) or "the separation between two things measured in units of length, or the length of a path joining two points" (Fyfield and Blane 1995, 70). Length is "the number of times a unit interval will fit in the line segment" (James and James 1992, 246) or "one-dimensional extent measured in units defined by a line segment" (Fyfield and Blane 1995, 125).[2] This may seem straightforward, but notice the nuance in the definition of *length* and consider that the line segment between the two points is no longer straight—therefore, the distance travelled by Michael's rabbit changes. We understood that the intent of the question was to learn whether or not a child could comprehend the notion of the conservation of length; that is, the length of the path that the rabbit travels does not change when the rods are angled.[3] But when Michael compared the angled rods to the straight rods, he could see that the rabbit did not travel as far on the desk. The straight rods got the rabbit further ahead on the desk. Michael saw it as kind of contest. In fact, in the assessment interview he said that the rabbit on the straight rods would win. This would indicate that Michael does not yet have an in-depth understanding of length and distance because he is attentive to the context. In fact, we do not know about the level of his understanding of length and distance because he is working so hard to help Raina understand the importance of the context.[4]

In the list of achievement indicators for Grade 1, an indicator of meeting the measurement outcome is "determine which of two or more given objects is longest/shortest by matching, and explain the reasoning" (Alberta Education 2007, 61). For Michael, the story of the rabbit trumps the comparison of the length of the rods. In relation to this achievement indicator, while Michael might be seen as struggling with the first part of the indicator, he is proficient at the second. His reasoning is sound.

The implication for educators is to be able to elicit knowledge from children about their understanding of concepts such as distance and length in ways that attend to the complexity of their thinking. The strength of the one-on-one assessment interview is that it allows us to more fully engage with the child's reasoning, which might mediate our evaluation of a child's knowing.

This research takes up the work of Dr Grayson Wheatley (Wheatley 1990, 1991, 1992, 2002; Wheatley and Reynolds 1999) and his deep interest in the complex thinking of children and the possibilities for encouraging this thinking in mathematics classrooms. We used an assessment instrument designed by Wheatley to be conducted in a one-on-one interview between a child and a teacher. We hoped that using this assessment interview would help us discover opportunities for children to demonstrate more complex mathematical thinking not typically found in paper-and-pencil assessments.

As we conducted this inquiry, we came to realize that the interview gave us a different understanding of the children and their mathematical knowledge. There have been many calls for teachers to differentiate instruction in order to meet the diverse educational needs of children. We began to realize that there is also a need to differentiate assessment. Differentiated assessment necessarily goes hand in hand with differentiated instruction. We awakened to this notion as we watched children and teachers work together in their assessment making (Clandinin et al 2006) during the mathematics interview. We saw how the act of conversing with children helped teachers know the children with whom they worked in more complex ways. This has implications for teaching styles and how teachers teach different mathematics concepts. Through the interviews, teachers reached an important realization of how they began to understand the children as knowers and sense makers.

Differentiated assessment does ask teachers to consider more in their work alongside children in mathematics classrooms. We are not suggesting that paper-and-pencil assessments be replaced, but, rather, that they are not the only way of understanding what children know. Wheatley's interview assessment was a tool we chose to use in our work. It is a multiquestion assessment that takes approximately 45 minutes, though using only a few questions derived from planning or a more traditional assessment might suffice. In fact, we can see how a planned conversation (Glanfield et al 2003) about parts of a paper-and-pencil assessment would provide a deeper understanding of the child and provide the teacher with more information to communicate to parents.

Notes

1 This inquiry was supported by a grant from the Dr Stirling McDowell Foundation for Research into Teaching.

2 As our research team discussed meanings of distance and length, we learned that many definitions of *distance* include the word *length*. We began to wonder about the ways in which we use these terms in our own practices and how each of us had come to make sense of the terms.

3 Our research team also noted that the manipulative used in this assessment task was Cuisenaire rods. The nature of the rods actually shows that the length of the path might change when the rods are angled, depending on where the rabbit travels on the rod. For example, if the rabbit travels down the middle of the rods then the rabbit would have to hop over a small gap between rods when they are angled.

4 Our research team also discussed the questions that Raina might have used to learn more about Michael's understanding of distance and length, and we discussed how Raina might use what she now knows about Michael's conceptions of distance and length to plan for future instruction. We do not include the discussion of these items in this paper, because the focus of the paper is on what can be learned from children about their mathematical knowledge in an assessment interview.

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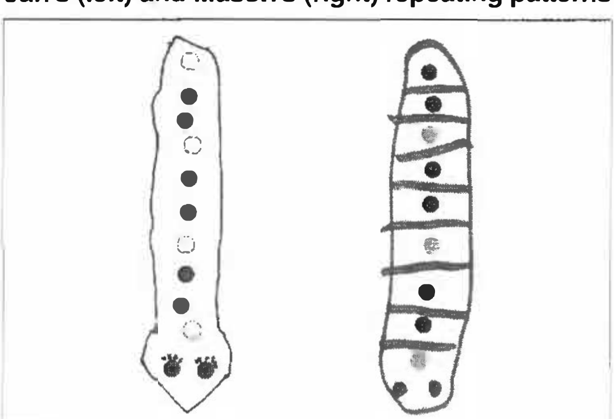
Assessing Young Children's Attention to Pattern and Structure

Lynn McGarvey

A mathematician, like a painter or a poet, is a maker of patterns.
—G H Hardy

Mathematics is sometimes described as "the science of patterns" (Devlin 1994; Steen 1988, 611). As Steen (1990) wrote, "Mathematics is an exploratory science that seeks to understand every kind of pattern—patterns that occur in nature, patterns invented by the human mind, and even patterns created by other patterns" (p 8). Across North America, mathematics curricula in the early years emphasize the importance of pattern as a way for children to make connections to the world around them and as the foundation for algebraic thinking (NCTM 2000). From pre-K to Grade 2, children are expected to recognize, identify, duplicate, extend and translate simple sequential patterns using a variety of attributes including sounds, actions, colours, shapes, objects and numbers. Early childhood classroom walls are often adorned with a variety of colour- and shape-patterning products. However, these products often don't reveal the range of mathematical reasoning that takes place when the patterns are made. For example, examine the patterns in Figure 1 created by Jun and Mason, both age 6. Both children have created a similar repeating pattern successfully and independently, but their reasoning about patterning is very different.

Figure 1:
Jun's (left) and Mason's (right) repeating patterns



Jun described her pattern as "yellow-blue-blue-yellow-blue-blue-yellow ..." and pointed to each dot on her snake. When asked to describe her pattern, she said, "There are two blues between the yellows." And when asked how many dots made up her snake, she pointed and counted, "1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and then 11, 12 for eyes." Jun has met many of the expectations for repeating patterns, and we might assume that she knows repeating patterns well, but the curriculum expectations do not provide a clear indication of what teachers should be looking for in children's descriptions and do not help teachers recognize the link to algebraic thinking that underlies patterning activities. Jun's interpretation of the pattern as "two blues between the yellows" makes it difficult for her to see the structure of the pattern as a whole.

Mason's response provides a contrast in experience and reasoning about patterns. As Mason was making his pattern (before the lines were drawn), he was asked to describe his pattern he said, "It's a red-red-green pattern. That's the core. Do you want me to circle the core?"

"No, that's okay. Just keep making the pattern for your snake."

"I could change it by putting a green dot at the beginning [tail] and make it a green-red-red-green pattern ... No, wait. It would just be a green-red-red pattern, but I'm just going to keep it [as red-red-green]." He finished putting down his dots and I asked, "You used the word core. How many times does the core repeat?"

"Three."

"Do you know how many dots you used for your pattern?"

"Uh ... nine."

"Oh [expecting him to count]! How did you get that?"

"Well, I know that six and three is nine, so it was easy."

"Where did the six come from?"

"Two of these [two units of the core] are six and one more is another three. So nine."

Mason's description of his pattern, his identification of the pattern core, his flexibility in counting the core units as a group of three dots and then using that information to determine the number of dots altogether provide a solid basis for later understanding of multiplication, algebraic expressions and functional relationships.

This paper provides an example of a repeating patterns assessment task that can be used with children aged 4 to 8. The task and variations of it reveal children's reasoning about patterns. Four types of reasoning are shown to orient teachers' attention during the patterning process and also provide guidance for instruction. Although the content of the task uses repeating colour patterns—which are the simplest form of pattern and attribute—the task may easily be adapted for other repeating patterns (for example, border, hopscotch) with a variety of visual attributes (for example, shapes, objects).

Repeating Patterns Assessment Task

(adapted from Papic and Mulligan 2007)

Materials and Preparation

- Connecting cubes in six colours: Create a two-colour ABB tower (for example, yellow-green-green) (see Figure 2) and a collection of individual cubes in the same two colours, plus a third colour used as a distracter (for example, black). Create a second two-colour ABB tower in different colours (for example, orange-blue-blue) and a collection of individual cubes in the same two colours, plus a third colour (for example, white).
- Strips of legal size paper cut in half (that is, 5.5" × 14")
- Coloured dot stickers in three or four colours
- Markers

Set-Up

Working with pairs of students, give each child an ABB tower and coloured blocks (see Figure 2).

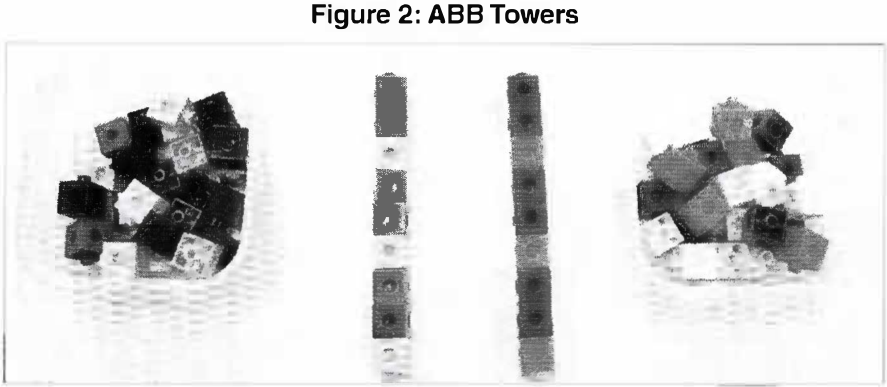
Assessment Task

The following questions represent many of the outcomes for patterns in the early grades including identify, describe, copy, extend, compare and create patterns.

1. **Identify/describe:** Give each child a premade ABB tower and ask, "Is it a pattern? Why do you think it is (or isn't)? Describe the pattern." For kindergarten, ask, "What is the part that repeats?" For Grades 1 and 2, ask, "What is the pattern core?"
2. **Reproduce:** Give each child a set of individual cubes (two of the correct colour and a third colour) and say, "Make a tower exactly the same as this one." Depending on the children's previous experiences, either leave the tower on the table for them to make comparisons (preschool to Grade 1) or show the tower for five seconds and then hide it (Grades 1 to 3). If they have difficulty, show it again for a few seconds. If they still have trouble, leave the tower out to be copied.
3. **Extend:** "Can you add more blocks to your tower? What would come next on the tower? How do you know that block comes next?"
4. **Compare:** Have the pairs of children compare their towers with each other and ask, "How are the two towers the same? How are they different?"
5. **Create:** Remove the blocks and give each child coloured circle stickers and a strip of paper. "Make your own pattern with coloured stickers."
6. **Identify/describe:** "Did you make a pattern? How do you know? Describe your pattern. What is the pattern core? How many times does the core unit repeat?" It may be helpful to have the child circle the core units with a marker.

This assessment task may be modified for a variety of materials and for the experiences of the children being assessed. The general goal of the assessment task is to understand the children's reasoning about patterns. Not every question needs to be asked, and

Figure 2: ABB Towers



modifications may be made depending on the child's responses. The next section provides a range of children's patterning strategies, from preschool to Grade 2, in response to aspects of the assessment task.

Children's Attention to Pattern and Structure

The assessment task is not a measure of understanding, but an indicator of how children perceive patterns and what strategies they use when working with patterns. The information gathered is intended to inform instruction. In this section, four types of responses are provided based on working with children from age 4 to 7. The range of responses is not intended to be developmental—that is, children will not necessarily go through each phase. In fact, children will attend to patterns differently, depending on the attribute. For example, children are often very successful with patterning tasks that focus on colour patterns, but they might have difficulty when patterns focus on shape, sound or other attributes. Differences in children's responses, such as those seen with Jun and Mason at the beginning of the paper, are due primarily to previous experiences and instructional orientation.

1. Inattention to pattern and structure

When asked, "What is a pattern?" Abed (age 4) did not have a definition or description. Not being able to define a pattern is not necessarily an indicator of understanding, so the assessment continued, and Abed was asked to make a copy of the orange-blue-blue tower he was given. Although I tried to encourage him to build the same tower, he either did not understand or was not interested. He was eager to build another tower, but he did so by randomly putting the blocks together (see Figure 3). When it got too long and started breaking apart, he began adding blocks to the original tower. The circle around the blocks in Figure 3 shows the original tower that remained intact. Abed appeared very motivated to build with the blocks, but he did not attend to the pattern as he did so.

2. Direct comparison strategy

Sophie (age 5) was given the yellow-green-green tower and was asked, "Is it a pattern?" She responded, "Yes," and described it as "yellow-green-green-yellow-green-green-yellow-green-green" as she pointed to each block in the tower.

"How do you know it is a pattern?" She responded, "Because it has yellow and green and they keep going."

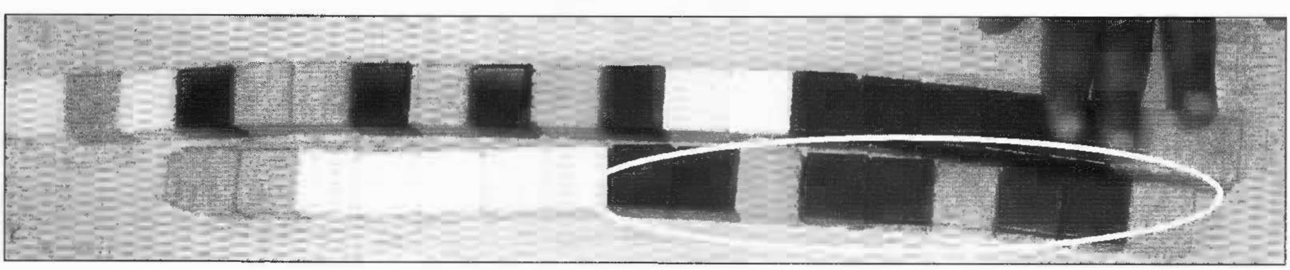
"Do you know what the core is?" She shrugged her shoulders.

"Here are some more blocks. I want you to make a tower exactly like this one, okay?" Sophie got a yellow block and then a green one and put them together. After this initial building she lined up her tower with the premade one to determine which colour would go on next. Her completed tower was identical to the original, but to examine her process more closely I created a revised task.

On a strip of paper I used yellow and green stickers to make a yellow-green-green pattern and asked her to make a copy. This time she also tried to use a direct comparison strategy by placing a finger on the original pattern at the left and putting a matching sticker on her pattern on the right. Once she had placed a sticker, she looked back to the original tower and found a dot just above the height of the sticker she had just placed to find the next sticker in line. Since there were longer gaps in her sticker tower, she missed one of the green stickers in the middle of the pattern.

Children who use a direct comparison strategy will often be able to successfully copy patterns when objects fit together; however, they have more difficulty when they are asked to copy a pattern with stickers or stamps or by drawing, because the spaces between elements can vary, and it is more difficult to line patterns up to make a direct comparison. Children using this strategy may say that the original and copy (like that in Figure 4) are the same by looking at it. It is only when they read the pattern and hear the verbal pattern breaking down that they are able to correct the pattern. For example, when Sophie read

Figure 3: Inattention to pattern and structure



her pattern, "Yellow-green-green-yellow-green-yellow," she heard the error and said, "Oh! I made a mistake."

3. Recursion strategy

Hua (age 6) described his tower as orange-blue-blue-orange-blue-blue-orange-blue-blue and said that it would keep going. He was able to copy and extend the pattern with blocks fairly easily. As he was building and extending the tower I asked, "How do you know which colour comes next?" He had just put an orange on and confidently said, "Blue comes next."

I asked, "And then what?"

"It's another blue."

"Then what?"

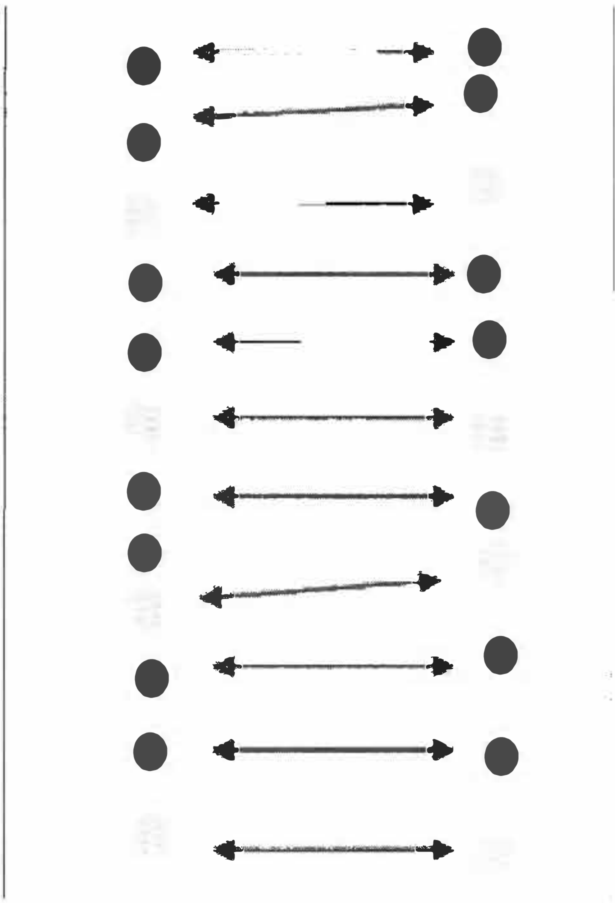
"Orange."

"So how do you know what comes next?"

"I look at this one [pointing to the last block put on] and then put the next one on."

Hua's response suggests that he is using a recursive strategy to build the tower. He knows what comes next by looking at the last block that was put on.

Figure 4: Original pattern (left) and Sophie's direct comparison process (right)



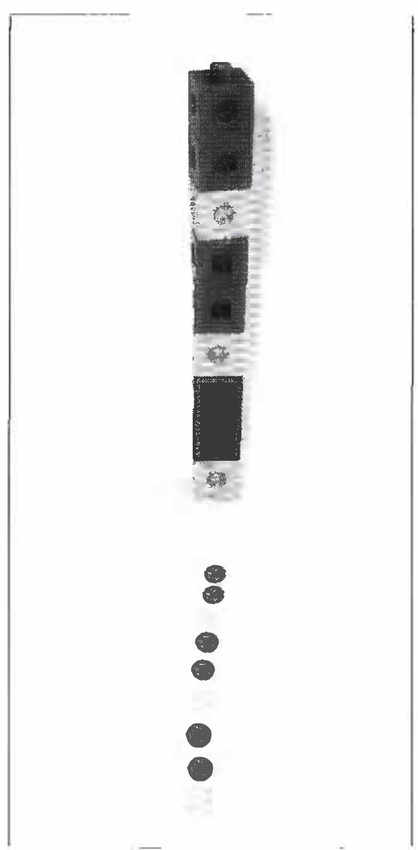
Children are often able to produce the expected pattern using a recursion strategy, but it is in the making and extending of patterns that the recursion process appears. A recursive strategy is used frequently by children (and adults), but it becomes less effective with more challenging patterns when the number of elements in the core unit gets longer, when the materials used are less familiar and when the shift is made to number patterns. For example, in the number pattern 4, 7, 10, 13, a child might use a recursive strategy of plus 3 to determine that 16 comes next. However, the only way the child can determine, for example, the tenth number in the pattern using a recursive strategy is by adding 3 until the tenth number is reached.[1]

4. Core unit strategy

I showed Reagan (age 7) the yellow-green-green tower for three or four seconds and then put it behind my back and asked her to make the same pattern with stickers. She quickly and easily placed the stickers on the page.

"Wow! That was fast. How did you know how to build the tower?"

Figure 5: Reagan's copy of tower using pattern unit strategy



Reagan said, "I remembered yellow-green-green [core unit] and there were three of them [units]."

If a child can see the pattern core, she doesn't have to remember every single block. Reagan showed that she needed to remember only the core unit and how many units there were. Looking for a core unit allowed Reagan to look for a relationship between the number of times the core unit is repeated and the number of elements in the core. Reagan demonstrated flexibility in being able to count with units other than one. A core unit strategy is also directly related to identifying a relationship in a function. A functional approach allows a person to determine any number of elements in a pattern without having to know all of the numbers in the sequence.

Conclusion

Human beings are naturally inclined to make sense of their environment by searching for patterns in images, objects and events. While early patterning activities might produce pretty pictures for classroom walls, supporting young children's understanding of patterns provides an excellent starting place for mathematical thinking. This paper provides an example of an assessment task, but the questions asked during the task are also important for daily instruction in patterns:

- Is it a pattern? Why do you think so?
- How are the two patterns the same? How are they different?
- What is the pattern core? How many times does it repeat?

Instruction needs to draw children's attention to what is and what is not a pattern, finding similarities and differences in patterns and the structure of patterns by attending to the pattern core. Our assessment of children also needs to shift, from the patterning products that children produce to the reasoning and strategies they use in the process of copying, extending, comparing and creating patterns. Without a shift in our instruction and assessment, many children will continue to be successful in the outcomes related to patterns by focusing primarily on the repeating elements in a pattern (for example, red-green-red-green), but an understanding of patterns requires attention to the core unit that repeats (for example, red-green repeated three times). Understanding units and flexibly counting and comparing units is essential in many topics in mathematics, including place value, measurement, fractions, multiplication and unit circles in geometry. Patterns are at the heart of mathematics and mathematical thinking. Early childhood educators have the opportunity to help children see mathematics as the science of patterns, rather than just as exercises in counting and computation.

Note

1. The more efficient alternative is to determine a functional relationship. In the example of 4, 7, 10, 13, the function rule is "times 3 plus 1." The tenth number would be 10 x 3 + 1 = 31.

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Should We Teach Mathematics to Young Children?

An Awareness of Toddlers' Mathematical Learning Through Pedagogical Documentation

Kimberly A Gravel

Research with infants has demonstrated their ability to recognize and discriminate among small numbers of objects (Clements and Sarama 2009; Cross et al 2009; Varol and Farran 2006). Other studies have revealed the use of complex and sophisticated mathematical ideas emerging in children's play and everyday moments (Clements and Sarama 2009; Ginsburg, nd). These everyday experiences are the beginning of young children's interest in and understanding of the world from a mathematical perspective. However, this capability and the resulting opportunities for young children to learn and become competent in mathematics are, for the most part, not currently recognized or achieved in many early childhood and child care settings. How, then, can I bring an awareness of early mathematics to my work in child care with infants and toddlers? In this paper I will describe how, through the use of pedagogical documentation, I gained insight into children's mathematical learning.

Pedagogical Documentation Project

> Pedagogical documentation ... is mainly about trying to see and understand what is going on in the pedagogical work and what the child is capable of without any predetermined framework of expectations and norms. (Grieshaber and Hatch 2003, 90)
>
> It is through pedagogical documentation that learning processes can be shared, discussed, reflected upon and interpreted—not only by educators, but also by children, parents, and anyone wishing to gain deeper understanding. (Rinaldi 2005, 17)

From two rounds of investigation and using a pedagogical documentation inquiry process of framing and reframing questions, planning a starting point for the investigation, collecting data and analyzing data (Gandini and Goldhaber 2001), I endeavoured to learn about our toddler children's mathematical learning in their play. For two mornings I worked as part of a team with two other early childhood educators. Through deliberate and careful selection of data (pictures, my interpretations, educational quotes and teacher–child dialogue) that gave us insights into the children's sense making and mathematical learning, we developed a documentation panel and displayed it in the common area of our centre for families, visitors, staff and children to view. Documentation in this form allows children's learning to be made visible and brings together the educator's reflective interpretations on children's developing theories—what the children know and what they are learning. A teacher engaged in pedagogical documentation shifts from teaching children to studying and learning with the children. What follows is my learning story, a pedagogical documentation of what took place over two days of engaging in learning activities with a group of toddlers.

My inquiry began with an understanding, gained through research and literature, that children's play and interests are often the source of children's first mathematical experiences and that "these experiences become mathematical as the children represent and reflect on them" (Clements 2001b, 272). This understanding also implied that early mathematics is more than getting children ready for school or accelerating them toward elementary school math. "Appropriate mathematical experiences challenge young children to explore ideas related to patterns, shapes, numbers, and space with increasing sophistication" (*Early Childhood Today* editorial staff 2002, 1). Therefore, the beginning questions to my investigation were (1) where do we see mathematical learning in the children's play? and (2) how do our toddlers experience and make meaning of mathematical concepts in their play? Inspired by the following words from an interview of Clements (Clements 2001a), I began observing and engaging with a group of seven toddlers.

> When it comes to developing mathematical skills, the younger the children are the less we need to interfere. There's nothing to lose and everything to gain by putting on your "math glasses" as you watch children involved in activities all around the room. Try to understand what is really going on and then ask questions or offer objects that will help children see the math behind the activity. (p 6)

Day One of the Inquiry

With this inspiration to see through "math glasses," it was illuminating (perhaps even surprising) to see the abundance of mathematical content and learning experiences that became visible in our toddler children's play, daily activities, and child–teacher communications. Mathematical words and language appeared in meaningful context in many routine care activities, transition times and play experiences. For example, the early childhood educators modelled and extended math concepts in language communications at breakfast time—the math concept of one-to-one correspondence was observed in teacher verbalizations such as "One piece of toast for you, one for you and one for you." Similarly, the children's simple verbal communications demonstrated their developing understanding of the concept of addition by saying simply "More" to request or acknowledge another piece of toast or glass of milk (Clements and Sarama 2009). Transition and cleanup times also contained critical learning experiences and language of concepts of spatial reasoning, comparison and classification when the teacher said, for example, "Go put your facecloth in the laundry bucket," "Let's put the little blocks in the box and the big blocks on the shelf" (see Figure 1).

> Many types of language are vital for mathematics learning. One is simple mathematical vocabulary. The words *more* and *another* are among the first that toddlers learn. Indeed, young children's early language comprises many words that refer to quantity, shape, location, and the like, such as *one*, *only one*, *the most*, *round*, *straight*, *in front of*, *behind*, *underneath*, *big*, *bigger* and *biggest*. Now you do not usually think of these words as mathematical, but they are: they refer to fundamental concepts that mathematics formalizes and clarifies. (Ginsburg 2009, 409)

Clements and Sarama's Learning Trajectories Approach to mathematics (2009) is frequently recommended for teachers of children under the age of three to provide rich sensory and manipulative environments as the instructional focus. Therefore, activities that were purposely made available for the toddlers on day one of our investigations were block play, manipulative-type toys (stacking cups, wire bead mazes) and sand play. Music, songs and stories that included both actions and repetitive words were also planned activities. Along with two other early childhood educators, I observed and was actively engaged in the children's play to appropriately draw attention to and extend their experiential learning of mathematical concepts and language. I had learned to recognize and enhance early mathematics learning in a graduate class on children's mathematical learning, and I passed this on to my team. Our goal was to see the children's everyday play through a new mathematical lens and to enrich and extend that learning.

During free play time, a 25-month-old boy was solely engrossed in play with blocks (see Figure 2). He began by gathering his blocks and stacking one

Figure 1. Spatial, comparison and classification mathematics language in everyday activities

block on top of another. When his block tower reached just the right height he greatly enjoyed pushing the tower over and watching all the blocks tumble to the floor. The teacher said, "You really like watching your tall tower fall!" After observing this child's play for some time, the teacher offered an extension to this boy's play by asking, "What would your tower look like if you started with two blocks?" The teacher modelled the two-block tower and the boy imitated it.

What do we know about children's play with blocks and what have we learned from this child's experience? Clements and Sarama (2005) offer us some insight into children's block play:

> Infants show little interest in stacking. Stacking begins at one year, when infants show their understanding of the spatial relationship *on*. The *next-to* relationship develops at about 1½ years. At two years, children place each successive block on or next to the one previously placed. They appear to recognize that blocks do not fall when placed this way. Children begin to reflect and anticipate. (p 8)

Documenting this boy's play provided us with insights into his developing spatial skills, his understanding of how shapes combine and his knowledge of height and quantity (Clements and Sarama 2009). He also illustrated an understanding of relationships as he demonstrated that he knew how to stack his blocks to build a tall tower without it falling over. He also had an understanding of predicting outcomes, because he knew what would happen with just one push of a block on his tall tower. This boy's play demonstrated that "the benefits of block building are deep and broad" and that "children increase their math, science, and general reasoning abilities when building with blocks" (Clements and Sarama 2005, 7). In this play situation, the teacher understood and recognized the mathematical learning embedded in this child's play and was able to offer both mathematical language and an extension to the learning experience.

After morning free play, the sand table was made available to the children (see Figure 3). Shovels, spoons, small and large buckets, and ice cubes were added to the sand table to enrich and extend the children's play experiences. The teacher gave exploratory prompts and questions such as "Can you fill the large bucket with sand? How many small cups will fill the big bucket?" to draw the children's attention and play toward particular math concepts. Sand play offers many opportunities for exploring mathematical thinking, reasoning and concepts (Clements and Sarama 2005). The concept of measurement underlies the play—we observed the children filling a larger container using smaller cups or shovels over and over again. "Heavy," said a 22-month-old boy as he lifted a large buck filled with sand. "That's right," said the teacher. "Your big bucket filled with wet sand is heavy."

Following sand play was circle time, during which the teachers and children engaged in singing and acting out this song (see Figure 4):

> If your name is *Child's name*, *Child's name*, *Child's name*,
> If your name is *Child's name*, stand up now.
> Jump inside the circle, the circle, the circle,
> Jump inside the circle
> Then sit down.

Figure 2. Spatial thinking and reasoning in block play

The children listened with anticipation and, with their ability to predict, they knew that when their name was called it was their turn to stand up, jump inside the circle and then sit down. This simple action song contains both a repetitive rhythm and an action sequence, and thereby gives the children an early introduction to the mathematical concept of patterning. In order to achieve the action pattern the children were required to listen, recognize the relationship (Geist 2009), and repeat the pattern of standing up, jumping and sitting down.

Day Two of the Inquiry

> In any good math activity if you change the "variable" you change and expand the experience as well as the understanding" (Church 2001, 8).

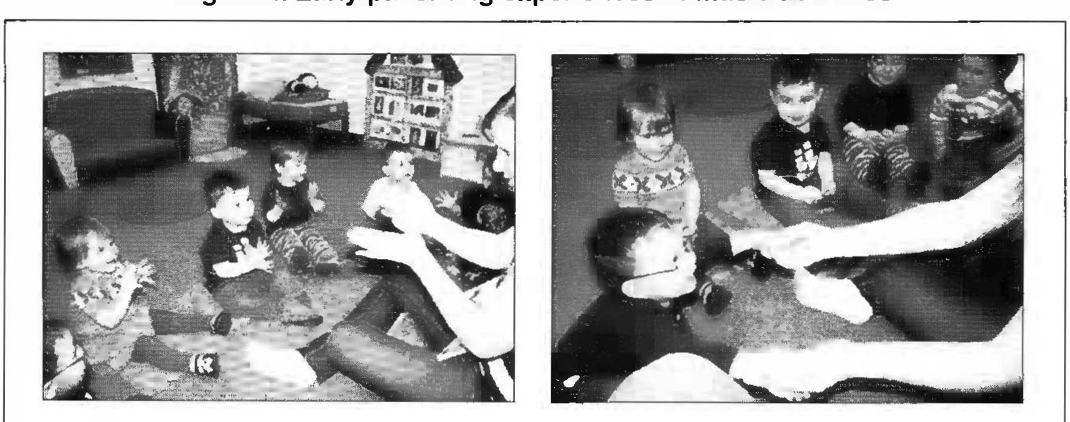
> Play does not guarantee mathematical development, but it offers rich possibilities. Significant benefits are more likely when teachers follow up by engaging children in reflecting on and representing the mathematical ideas that have emerged in their play. Teachers enhance children's mathematics learning when they ask questions that provoke clarifications, extensions, and development of new understandings. (Clements and Sarama 2005, 6)

With this understanding in mind and reflection on the first day of inquiry, the second day of investigations began. Questions leading this inquiry and observations were (1) what new play experiences can be provided to enrich and extend the children's mathematical learning? and (2) what math language can be taught to the children as they play?

On this morning, the teacher placed foam blocks on the playroom floor. The teacher sparked interest in this play by saying to the children "Look at all the different kinds of blocks and shapes we have today." During play with these foam blocks, the older children

Figure 3. Measurement in sand play

Figure 4. Early patterning experiences in music activities



quickly figured out how the rod-shaped tubes fit perfectly into the centre holes of the square foam blocks and the game became putting in and taking out these tubes. "Stuck," said a 22-month-old boy as he passed his square block to the teacher for help in removing the stuck tube. Two of the youngest children in the group (both 19 months old) began their play by merely watching (but with interested curiosity) their friends play the in–out tube game. The teacher recognized this and said to these two young toddlers, "Here is a square block and tube for each of you." Each toddler appeared to know what the other was doing and each tried to fit his tube into the circular hole. It took a few trials and errors, but after a little time both children successfully fit their tubes into the hole (see Figure 5).

By their documentation of this activity the teachers recognized and understood the meaning making of the children's in–out tube game. The children showed an incipient understanding of combining two parts (tube and block) to make a whole, which is an important early mathematics concept that is related to a later-developing concept of number composition (Clements and Sarama 2009).

Water play was the next activity that the teachers planned with the intention of having the children revisit the concept of measurement they had explored previously with sand. The teacher once again provided exploratory prompts and questions to draw the children's play and attention to particular math concepts in the water play: "Use your small cups to fill the big bucket with water," and "Here is a big bucket to fill with water" (see Figure 6).

Geist (2009) offers us insight into the children's meaning making during water play:

> Toddlers are still constructing the concept that simply changing the shape or arrangement of one or more objects does not change the quantity. This

Figure 5. Part-whole understanding with foam tube and block

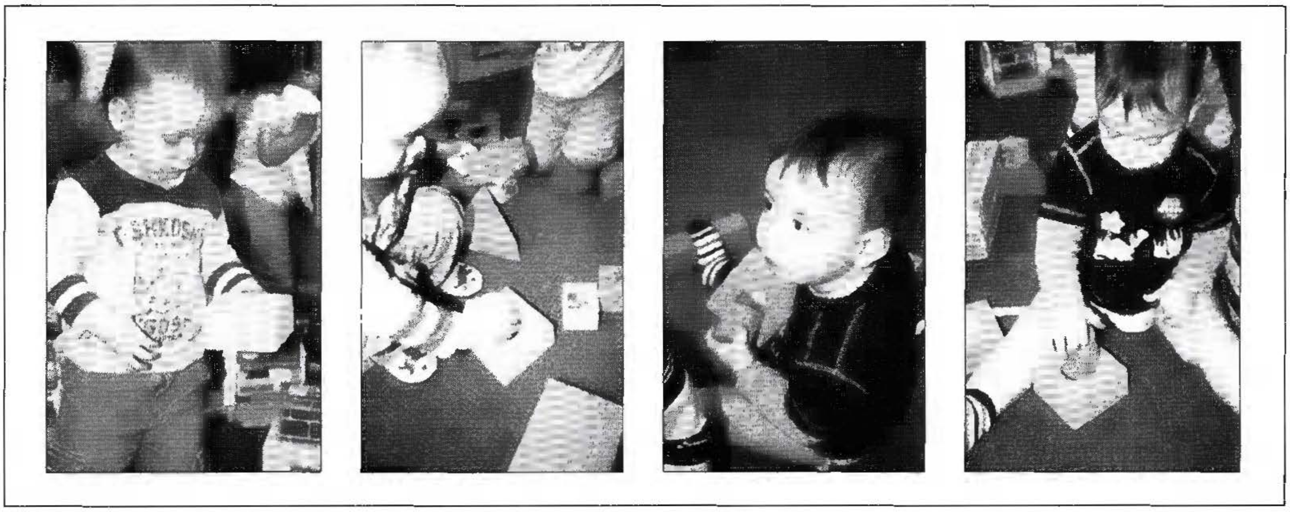
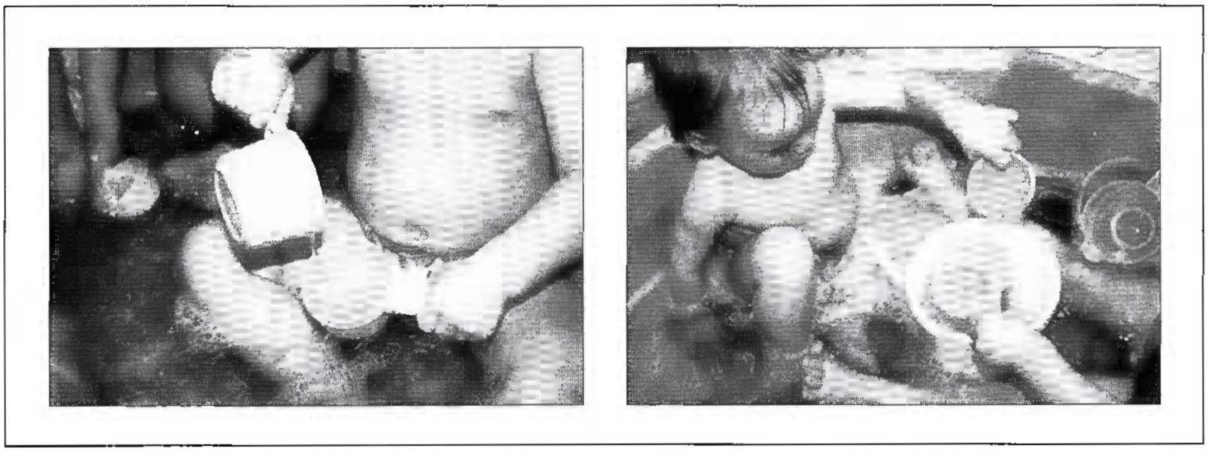


Figure 6. Measurement in water play



> understanding is known as *conservation*, and it will usually not begin to emerge until about age 4. However, this comprehension does not just pop into a child's head—it is constructed slowly, over time, as children play and interact with objects, containers, and substances such as sand and water. Conservation is important to future mathematical content areas, such as classification, seriation and number. (p 41)

Following water play it was time for gross motor play, in which we were going to move and explore the spatial concepts of *up, down, through, over* and *under*.

> Toddlers use their whole body to explore and learn. Being in different positions lets children pay attention to where things and spaces are in relation to one another. Physical activities introduce special relationships and set the stage for understanding geometry and numbers. (Geist 2009, 41)

We were intrigued to discover in this gross motor play the spatial concepts the children already understood and the spatial language they had. The teacher led a discovery walk through the playroom, which had been arranged with all shapes and sizes of climbing blocks. "Follow me," said the teacher. "Let's go on a discovery. We are going up—we are going down—we are going over—we are going under." The children watched the teacher moving through the path of climbing blocks and began their own exploration, talking as they moved. This activity provided surprising evidence that our young toddlers already possessed considerable knowledge and language of spatial concepts. The children moved up, down, over and through the play blocks and chanted, in a very natural way, "Up, down, up, down" as they explored and played. Even the younger toddlers who had not yet developed spoken language knew and could execute quite easily the spatial understanding of *over, through, up,* and *down* (see Figure 7).

Conclusion

Evidence from this pedagogical work illustrates that early mathematics learning can be developmentally appropriate, achievable and enriching for children under the age of three years. Readiness for mathematics is now understood by the educators in my centre not as a question of age-appropriateness but as a way to give children ample and enriching opportunities to explore and think about their world in mathematical ways (Clements and Sarama 2009). However, educators need knowledge about early mathematics content and learning, and they need the intentionality to teach, explore and support early math experiences. The process of pedagogical documentation was a means for educators to question, analyze, reflect, understand and share children's mathematical abilities.

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Figure 7. Spatial mathematics of *through, over, up* and *down* in toddler play

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For more than twenty years, Kim Gravel has worked in both daycare and after-school care programs. For the past eight years, she has been the executive director of the University Infant Toddler Centre, in Edmonton, Alberta. With her recent completion of a master's degree in education with a specialization in early learning and her experiences with young children, she brings together research and new insights to children's early learning and our work with these young children.

Subitizing: Part-Part-Whole of a Big Idea

Sylvia Malo

A continual cause for concern among many math teachers is how to help struggling students in our math classrooms. Some children who arrived in my Grade 4 or 5 math classes over the years struggled with number sense, and many came without the ability either to compose or decompose numbers or see part-part-whole relationships. The obvious breakdown in students' conceptual understanding of part-whole reasoning often occurred when students were asked to break up numbers into different parts and then recompose them or represent them in a different way. This was a common weakness for many students in my math classes and seems to still be an issue in many math classrooms today. After much reflection and searching, I have determined that one big idea in mathematics seems to provide more insight into why students struggle with number sense. This is my journey towards greater understanding of subitizing.

Subitizing in Relation to Counting

One of the most significant changes to my ideas of mathematical instruction in early years is the shift in emphasis away from the count as the first step to numeracy and quantifying. This change in focus is very difficult for many early childhood educators and parents to accept because it is a definite shift from the way we were taught as children. In the past 20 years of my career I was of the same mindset: students should count first, and then they can recognize quantity meaningfully—but this is not necessarily true. I was first inspired to look at a change in thinking regarding counting about three years ago, when I read an article by John Marshall in *Phi Delta Kappan* (2006), in which he stated

> Children come to understand these numbers as complete entities before counting. Counting comes after the numbers have been placed in order and when children know why three is more than two. Matching a set of three (cups) with two (saucers) in one-to-one correspondence will show which set has more members. (p 359)

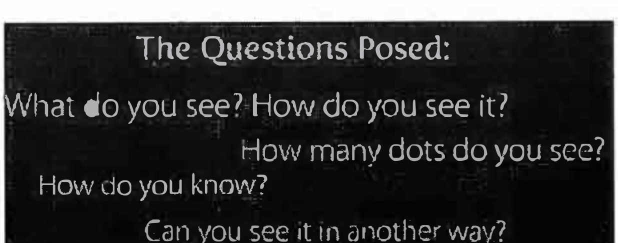
Marshall's claim made me rethink my understanding of early learners' capacity to identify number and quantity as well as question the importance of the rote count first versus matching, comparing and subitizing along with counting. Having worked primarily with older children, I had evidence that children who could readily count and read number words could not always quantify number. Many of the struggling students in my classroom over the years had had extreme difficulty breaking apart numbers (decomposing) and putting numbers back together (composing). I had incorporated some visualization or subitizing activities into a mental math portion of my lessons as starters. This visualization focus seemed to significantly change the way students saw and understood number. Clements and Sarama (2009) would identify this change as an incorporation of "conceptual subitizing—seeing the parts and putting together the whole" (p 9). It had a substantial impact on the older students' ability to visualize number as a composition of parts. This led me to pose yet another question: How could using subitizing activities with very young children affect their learning of number?

Recollecting my experiences with Division II students, I had observed that many of the struggling students could recognize regular arrangements of number, like those on dice or playing cards, but were unable to relate this and apply their recognition of number to irregular arrangements of dots. Students who had difficulties with the visualization or subitizing activities were often unable to apply or bridge the part-whole reasoning to operations like addition/subtraction or multiplication/division of whole numbers. It was not until the students actively engaged in subitizing activities and made explicit connections to the operations through the "flash method" of dot pattern cards (regular and irregular arrangements) and ten frames (Wheatley and Reynolds 1999) that these students in fact attended to number, quantity and number operations with greater understanding. By using the subitizing activities, some students began to see that numbers had more than one composite. Multiple compositions to produce whole numbers were possible. Students began to recognize different ways to compose and decompose numbers from whole to parts and from parts to whole. For example, 100 could be constructed in multiple ways using many different arrangements; a link to 10 was recognized by some students (4+6 could be recognized as linking to 40+60). Again, having had success with older children, I questioned why educators were not implementing these strategies at a younger age. Over the past few years I have witnessed more use of subitizing in the early primary levels—the Alberta K–9 mathematics program of studies (2007) now includes subitizing in the specific outcomes for kindergarten and Grade 1. I am anxious to see the effects of the implementation of this strategy longitudinally.

Details of Children's Learning from a Pedagogical Documentation: Subitizing

My participation in a University of Alberta early childhood mathematics course earlier this year strengthened my belief in the power and benefit of incorporating subitizing activities into young children's math lessons. In this course, I was asked to document classroom research described as a "pedagogical documentation." This process draws on Reggio Emilia practices in early childhood education. This form of documentation not only prompts teachers to think about children's work but encourages them to use the information to plan further activities with children. The project was intended to benefit the teachers, the children and the parents involved. I chose to study the effects of using subitizing with a group of 21 Grade 1 students in a rural area of northeastern Alberta.

The research project involved two rounds with students in both whole-class and small-group settings. Round one involved the whole class drawing what they saw when the image in Figure 1 was flashed. In round two, small groups of students were asked to participate by verbally responding to flashed images like the ones in Figures 2, 3 and 4. Round two also involved small groups of students drawing what they saw when the arrangements for 9 and 10 shown in Figures 3 and 4 were flashed.



Two students' responses in round one of the pedagogical documentation gave me some interesting insight and a good opportunity to view some students' ability or lack of ability to see part-part-whole relationships in arrangements of dots. These responses caused me to reflect on the limitations of students who have limited and/or emerging skills with subitizing and the ability to see parts in relation to whole number arrangements. Mary's[1] ability to subitize parts of the whole was evident—she saw 2s correctly to form 6 (see Figures 1 and 1a).

However, Mary could not answer without looking back at the picture she had drawn and counting each dot. I found this shocking. Mary proceeded to do this again with 9 in round two. She was able to subitize 3s in 9, but could not readily quantify the whole arrangement, 9, until she took part in the small-group discussion in round two. Mary frequently subitized parts as

Figure 1

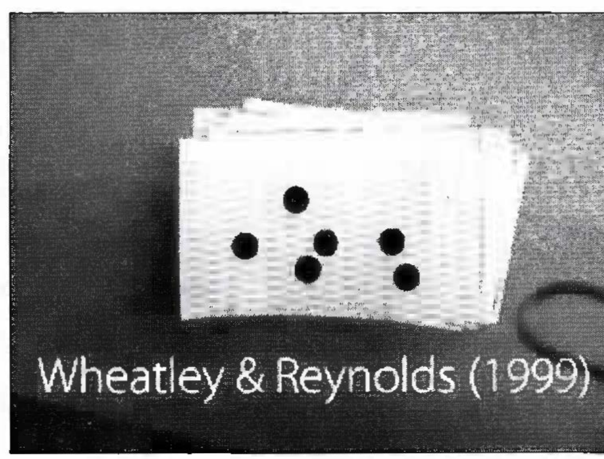
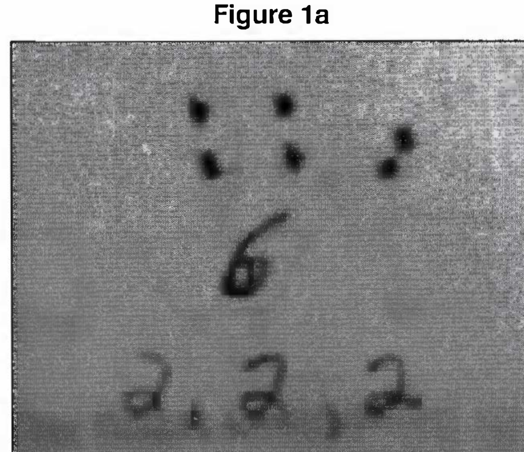


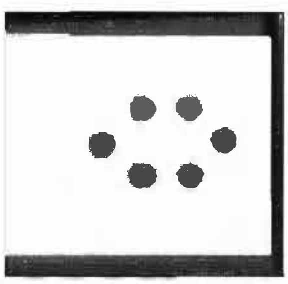
Figure 1a



2s or 3s, but only infrequently did she quantify without a count to verify the quantity. I would have to judge that this student's number sense was emerging; she saw the parts and was only beginning to relate the parts in relation to the whole arrangement.

Edward was another student that I thought was subitizing the arrangements in the first round of the pedagogical documentation. However, looking back at the video allowed me to recognize that Edward was sharing facts that he knew. He was not subitizing the images and could not recognize what parts made up the whole. This was proven to be the case in both rounds: in the first round Edward stated that he saw 7 for an arrangement of 6. This prompted me to ask, "How do you know?" He stated, "Two plus five equals seven," emphasizing his knowledge of this fact with five fingers and a counting-on action of two more. When I asked, "Where did you see the 5?" Edward replied, "On the bottom" (referring to the bottom of the dot arrangement—he was not able to readily identify where the 5 was). Even after I showed Edward the image again, he could not identify the quantity or parts for the arrangement of 6 that was shown (see Figure 2).

Figure 2



Edward's actions and statements illustrated the idea behind the quote/question posed by Hunting (2003), "Are finger sets used to represent visualized material, or simply used as a standard symbol set because visualization alone was too great a cognitive task? This we do not know" (p 232).

In Edward's case, I believe that his finger sets represented rote learned facts that he knew and that did not accurately reflect what he was seeing in the dot patterns. Edward's inability to see parts of the whole unit was again evident in the second round with his drawn images for 9 (see Figures 2a and 2b). He repeatedly attempted to change the images he had drawn after the irregular dot images were repeatedly flashed; however, he was not successful in making correct notations of the parts that he saw in the image or the whole arrangement of 9. Edward could not see parts of 9 or verify the quantity of the whole arrangement of 9, and he stated that he was "not sure" how many dots were in the irregular images. As evidenced in round one and again in round two, Edward knew some math facts (2+5 = 7) and could draw the regular square geometric arrangement of 9, but conceptually he did not see 9 as a composition of parts and a whole within irregular arrangements the dot patterns.

Visualizing and Verbalizing—Is It Enough?

Verbal responses alone did not provide enough information about what and how the students saw the dot patterns. Having the students draw the images and talk about their perspectives in round two provided much more information about student accuracies or inaccuracies of the subitized images. Having the students talk about their perspectives allowed me to note what and how they were seeing the parts in relation to the whole after they had created their personal drawings. For some students, just seeing the arrangement was a simple task—they "just saw" the arrangements for the smaller numbers. For these same students, combining the parts of an image to form the more complex arrangements was also a simple task. However, for others, seeing the parts required multiple viewings (up to three) and even with multiple views these students struggled with breaking up the irregular arrangements of 9 or 10 (see Figures 3 and 4).

Figure 2a

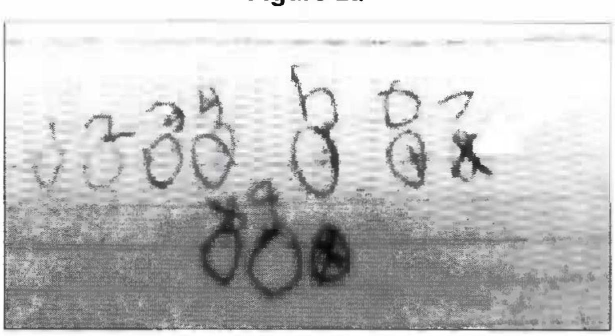


Figure 2b

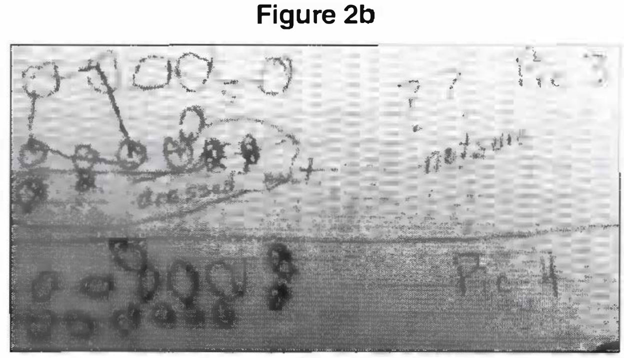


Figure 3

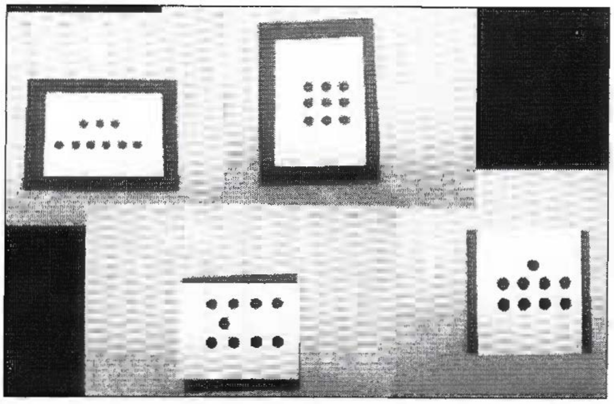
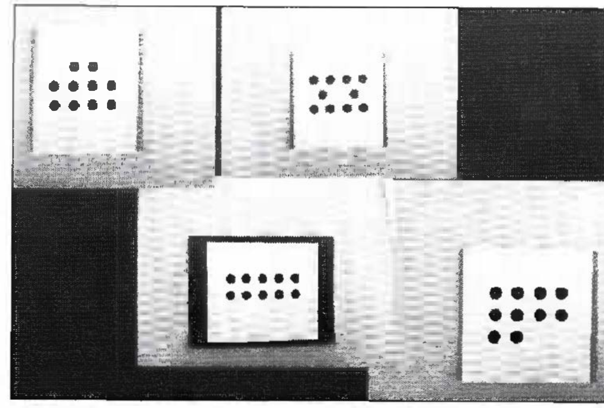


Figure 4



For other students, attending to the arrangement was a combination of seeing and drawing the parts to form a whole picture—recognizable shapes identified in the image helped some of these students see and then successfully draw the arrangement. Adapting this activity to have manipulatives available for the students to create, as opposed to drawing, the image may alleviate some of the frustration that some students had with the motor skills required to draw the image.

Content Analysis of the Mathematical Concepts: Subitizing—Where Do We Go from Here?

This pedagogical documentation dealt with the importance of subitizing and its role in students' ability to identify parts of a whole unit. What came to light were issues of students' inaccuracy in quantifying number even if they could accurately identify parts of the whole. This was also new learning for me. Prior to the pedagogical documentation, I had held the belief that if students could accurately subitize parts of a whole they could simultaneously quantify the whole. Additionally, I learned that students who could quantify the whole image did not always see the parts that made up the whole. This pedagogical documentation highlighted a key understanding: to be more successful in mathematics, students need to be able to both subitize parts and quantify number as a whole. Hunting (2003) alludes to this connection as well as an extension to mathematical operations:

> We are aware of the dynamics of part-whole reasoning where a subset is cut out from the whole while the whole set is kept in focus. We suppose that the logical operations of class inclusion are important here. The reverse situation, where a whole is rendered a part, by the conjoining of other items to make a new enlarged whole, prefigures the symbolic statements we know as addition of whole numbers. (p 232)

After completing this pedagogical documentation, there are three additional questions that require further exploration with young mathematics learners:

- Will just isolated work with conceptual subitizing dot patterns be enough to improve student ability to see part-part-whole relationships?
- Which subitizing activities make the most impact on student conceptualization of part-part-whole relationships as a link to number operations?
- How important are student discussion, discourse and sharing of personal perspectives to improving conceptual subitizing of number?

Conclusion

Pedagogical Documentation: Subitizing was an extremely powerful learning experience. The project brought to light several important issues and personal misconceptions around conceptual subitizing. First, the documentation reinforced the importance of children having multiple experiences with conceptual subitizing, with both regular and irregular arrangements of dot patterns for number. Surprisingly, accurately subitizing the dot patterns did not automatically mean that students could accurately quantify the arrangement they were seeing. Additionally, quantifying the whole did not mean that students could see parts of the whole. Students need to engage in active discussions about what they see and how they see the arrangements. Discussing their perspectives with peers and pointing out how they are comparing and combining parts to the whole are powerful. Actively encouraging student discourse in the math classroom allows children to develop alternative perspectives for the abstract arrangements, further highlighting the possible different arrangements of the parts in relation to the whole. Last, observing children and reflecting on their learning are powerful experiences that help us modify and adapt pedagogy to best suit student learning in mathematics. All in all, this was an incredible experience for me as a teacher in the role of teacher/researcher.

Note

1. Students' names have been changed throughout to protect privacy.

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